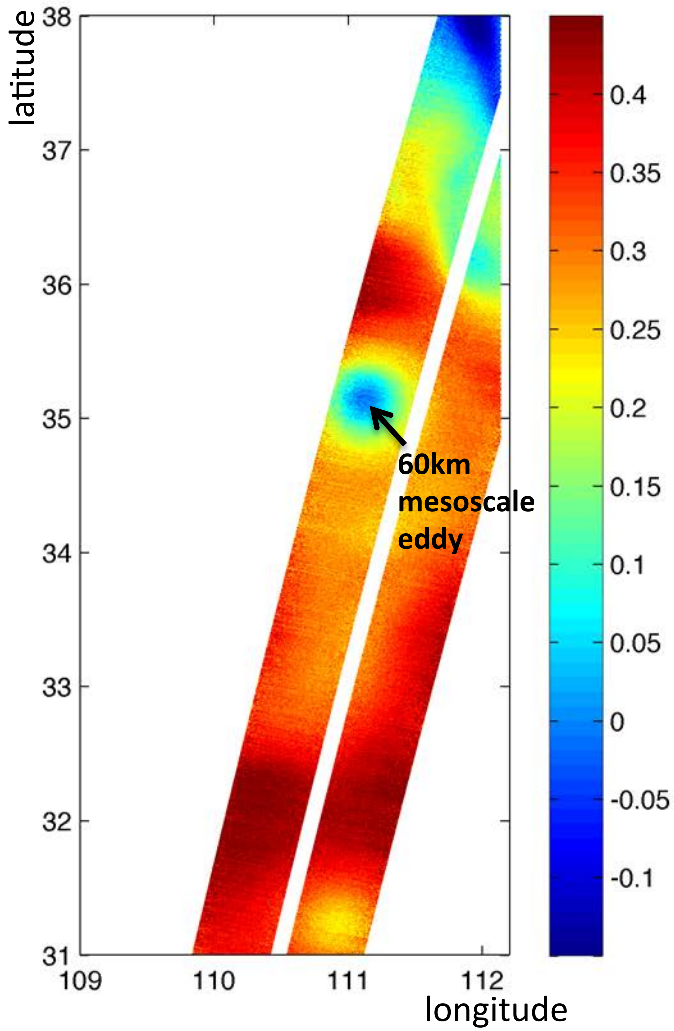
High-level products and reconstruction techniques for SWOT

Clement Ubelmann

Contributions from Lee-Lueng Fu, Patrice Klein, Bruce Cornuelle, Gérald Dibarboure, Marine Rogé, Rosemary Morrow, Lucile Gaultier

Mapping the future SWOT data?

SSH sampled with SWOT
(from high-resolution simulations)



- High spatial resolution on the swath

- Poor temporal resolution

→ How can we resolve SSH(x,y,t) from SWOT ?

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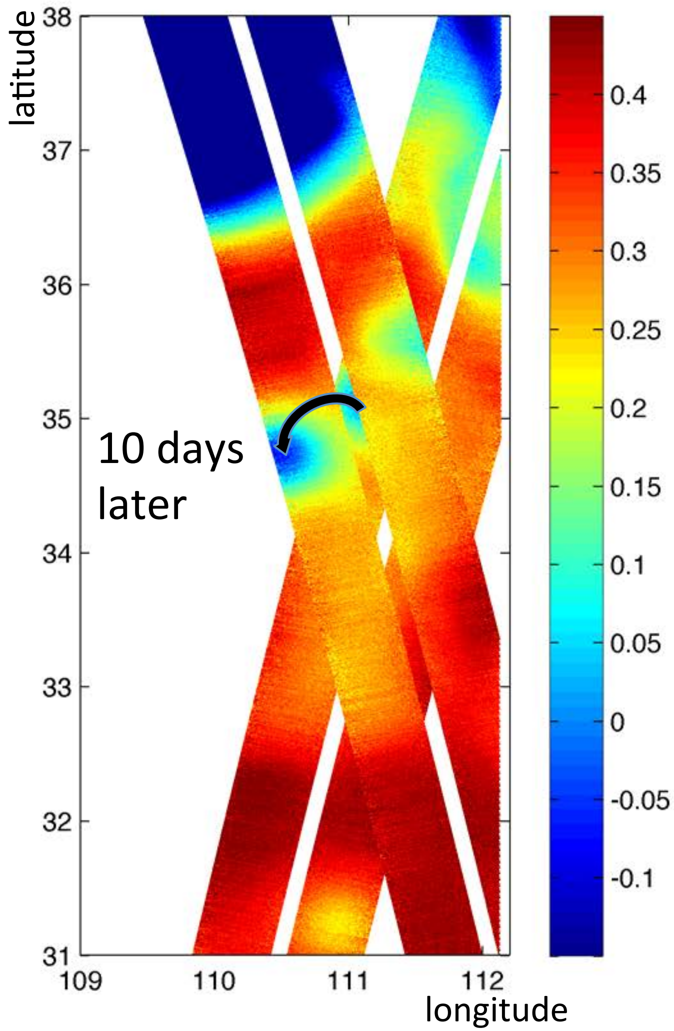

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→ How can we resolve SSH(x,y,t) from SWOT ?

Outline

- The present mapping (L4) method in altimetry and how it would work for SWOT?
- Why going further linear analysis?
- Exploration of a dynamic interpolation method: results, limitations, future improvements …

Objective analysis

Slide from G. Dibarboure

- Objective analysis (optimal interpolator)

$$H(r,t) = \sum_i \sum_j A_{ij}^{-1} C_{rj} H_0(r_i)$$

Along-track Observations

Grid pixel

Covariance between observations

Covariance between grid pixel and observations

- Covariance matrices describe the properties of mesoscale and measurement errors
- Example of covariance model for mesoscale:

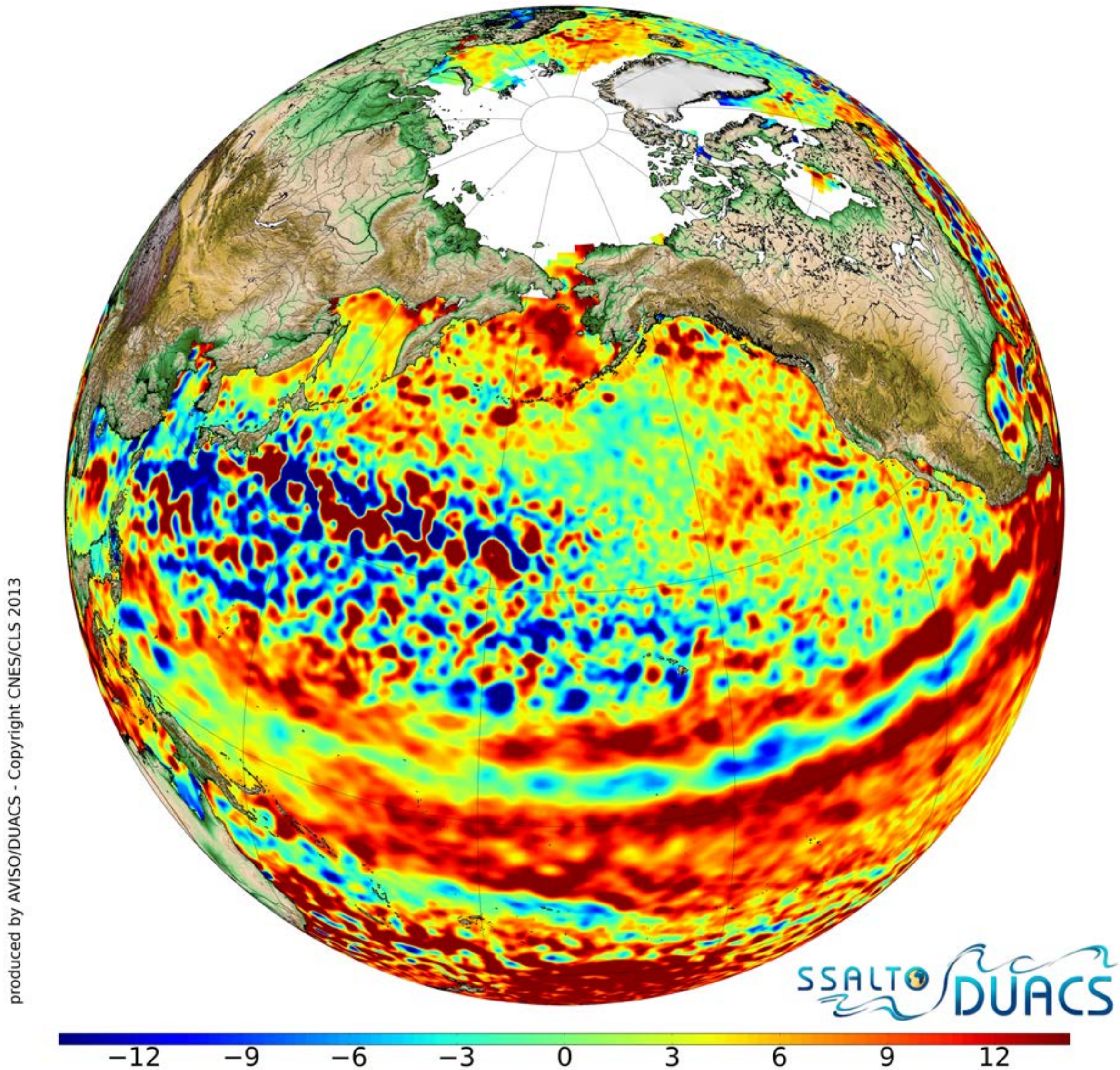
$$\langle h,h \rangle = \mathrm{var}(h) * C(r,t) \qquad r = \sqrt{\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2}$$

$$C(r,t) = \left(1 + r + \frac{1}{6}r^2 - \frac{1}{6}r^3\right) * e^{-r} e^{-\left(\frac{t}{t_0}\right)^2}$$

Arhan et Colin de Verdière, 1985

- The operational model also accounts for propagation velocities and x0, y0, t0, vx0, vy0 are fitted on along-track data and adjusted to ensure homogeneous sampling

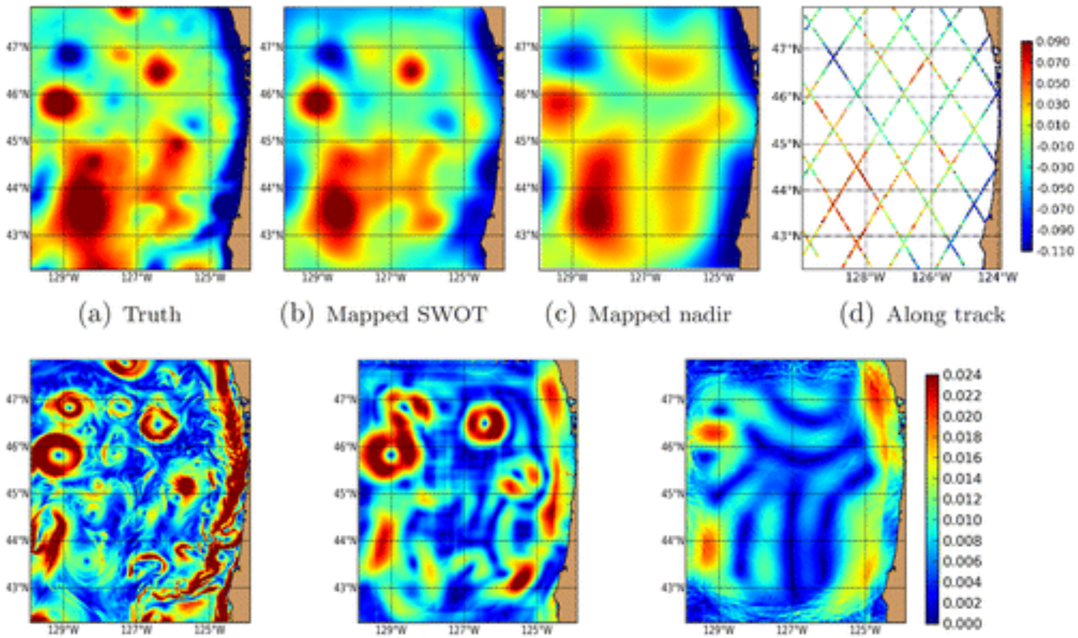
Today's AVISO maps



Merged Aviso map from AltiKa, Cryosat, Jason-2 and Jason-3

Objective mapping of SWOT data

From Gaultier et al., JTECH, 2016



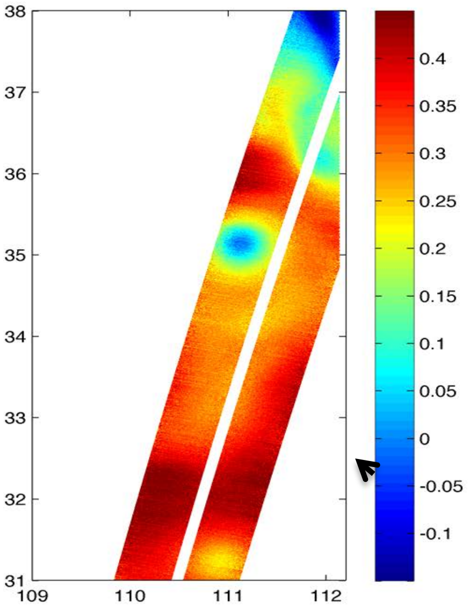
(a) Truth (b) Mapped SWOT (c) Mapped nadir (d) Along track

(e) True Velocity (f) From SWOT-like mapped observations (g) From Nadir-like mapped observations

- The method applies with a few technical adjustments:
  - «super obs» for matrix inversion of reasonable size
  - Or inversion in reduced parameter space.
- **SWOT mapping can preserve ~80km eddies in favorable situations.**
- **Important loss between SWOT subcycles. Eddy displacement not anticipated by the linear model of covariances**

The mapping challenge

Is a linear model still appropriate to describe error covariances for SWOT ?



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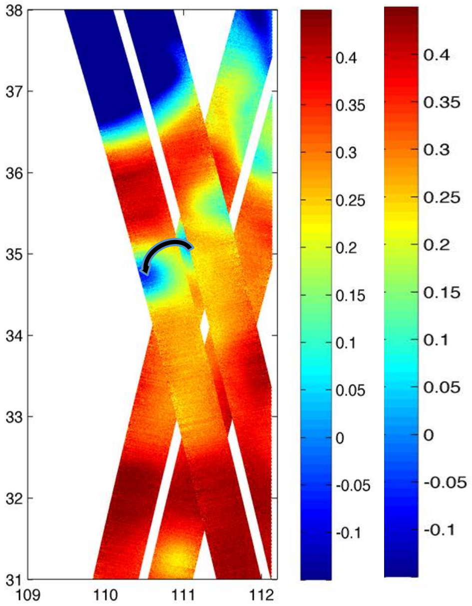

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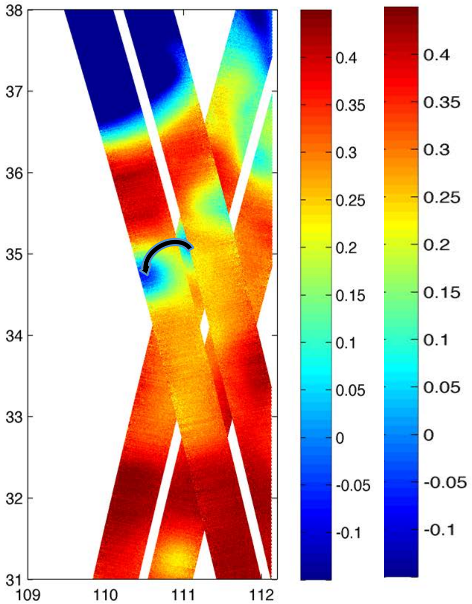
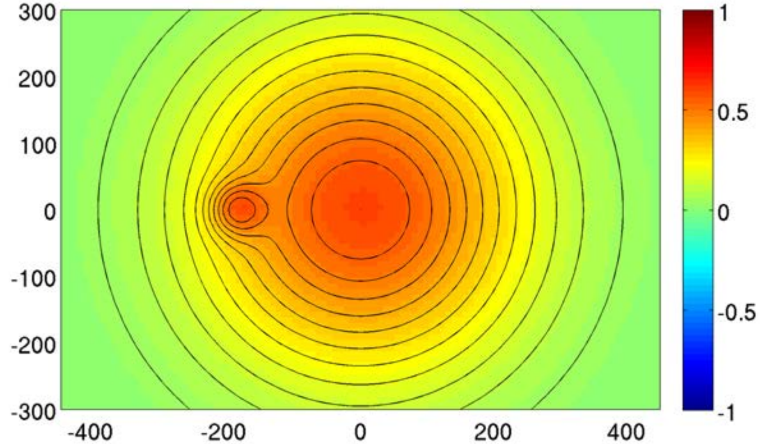


Strong non-linearities: active advection of PV



$$\psi = \frac{g}{f} SSH$$

$$q = \nabla^2 \psi - \frac{1}{L_R^2} \psi$$

$$\frac{\partial q}{\partial t} + J(\psi, q) - \beta \frac{\partial \psi}{\partial x} = 0$$

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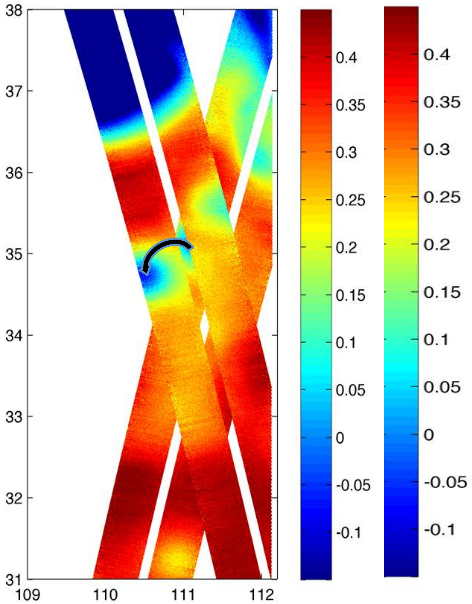

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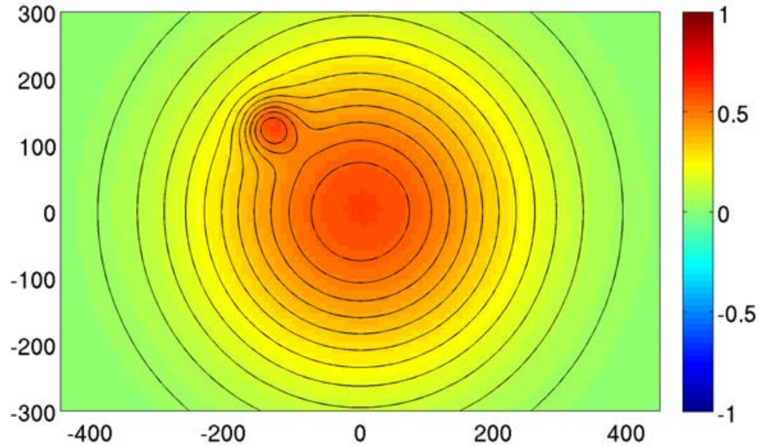

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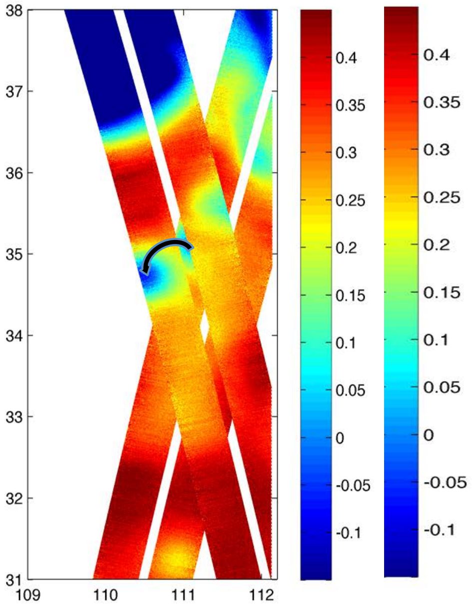

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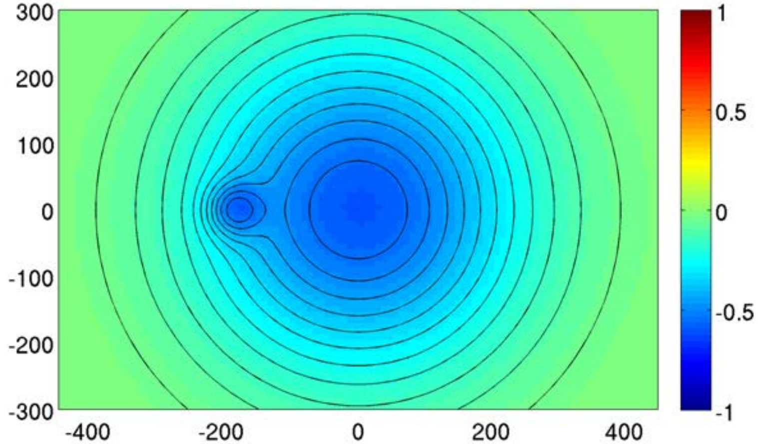

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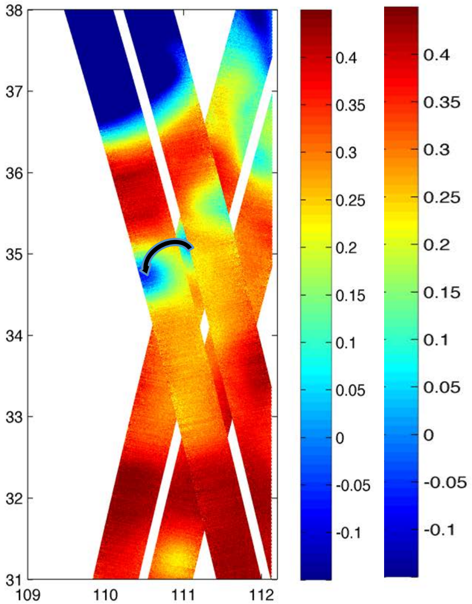

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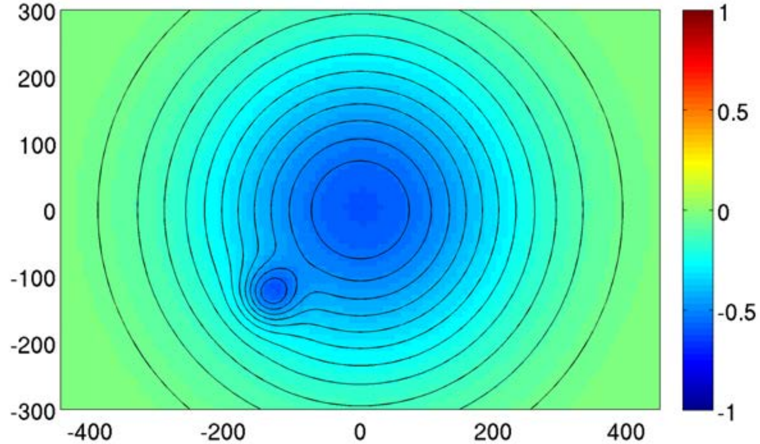

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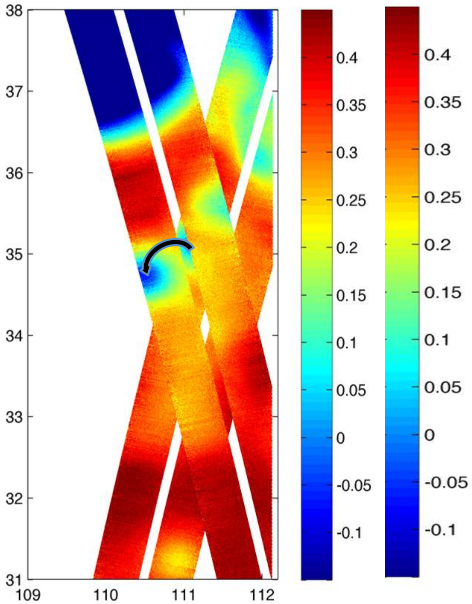

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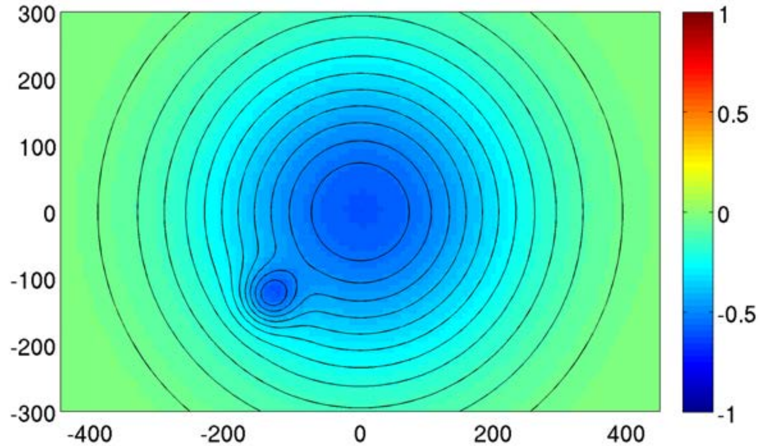

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$$\frac{\partial q}{\partial t} + J(\psi, q) - \beta \frac{\partial \psi}{\partial x} = 0$$

Today's OI mapping cannot handle non-linearities :

→A significant part of the SWOT signal would be filtered out of the maps, unused.

Explore beyond linear OI to design new data products

One approach explored: dynamic interpolation

- To account for some non-linearities of the eddy motions

- To provide an intermediate solution between statistical mapping and data assimilated products: use of reduced models

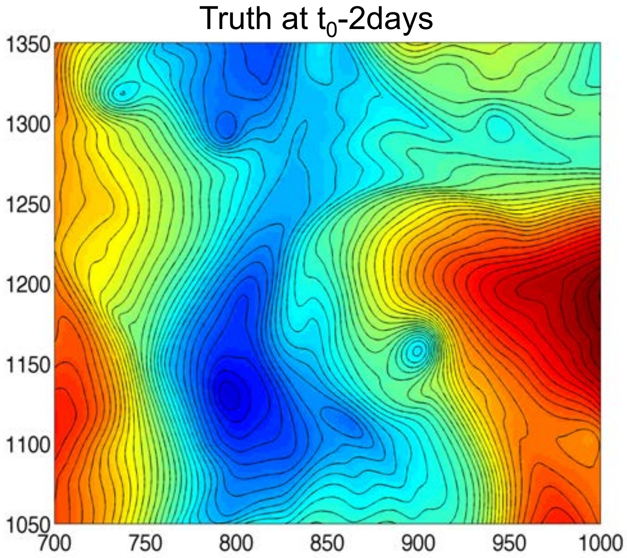
$$\psi = \frac{g}{f} SSH$$

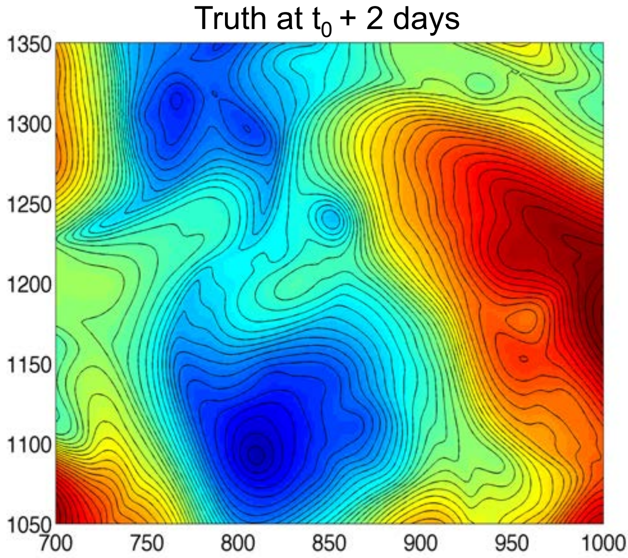
$$q = \nabla^2 \psi - \frac{1}{L_R^2} \psi$$

$$\frac{\partial q}{\partial t} + J(\psi, q) - \beta \frac{\partial \psi}{\partial x} = 0$$

Short-term propagator of the SSH field to help filling the SWOT gaps

Illustration on a simple case





More details in Ubelmann&Klein&Fu, JTECH, 2015

- The use of the propagator significantly reduces residual errors

→ Simplified illustration (no instrument noise, full snapshot) showing the potentials of reduced models

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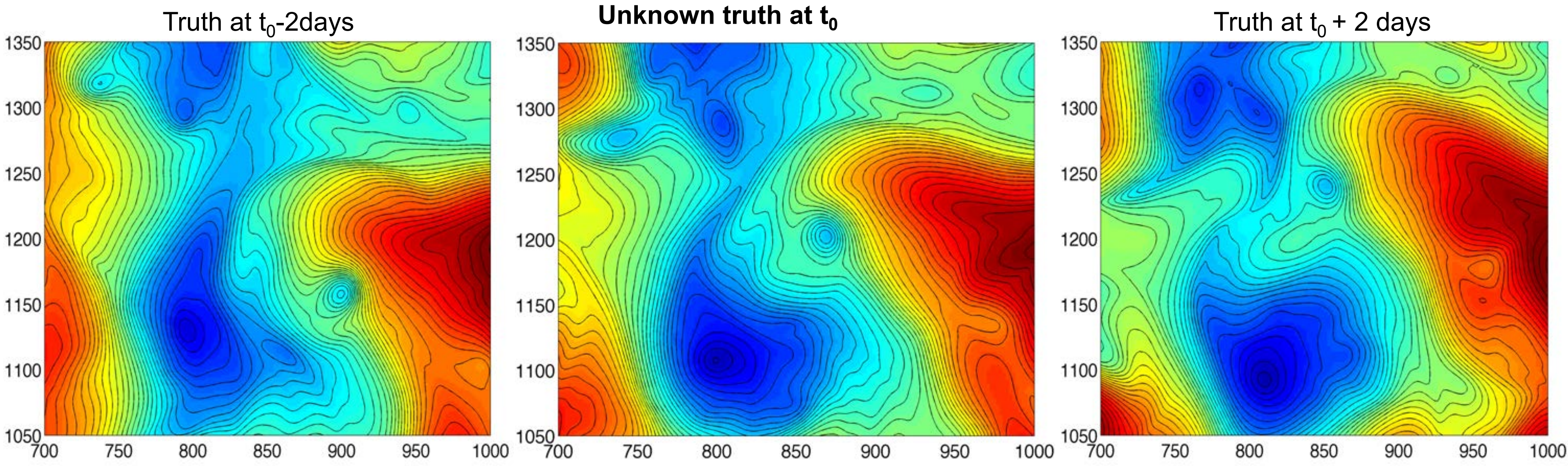

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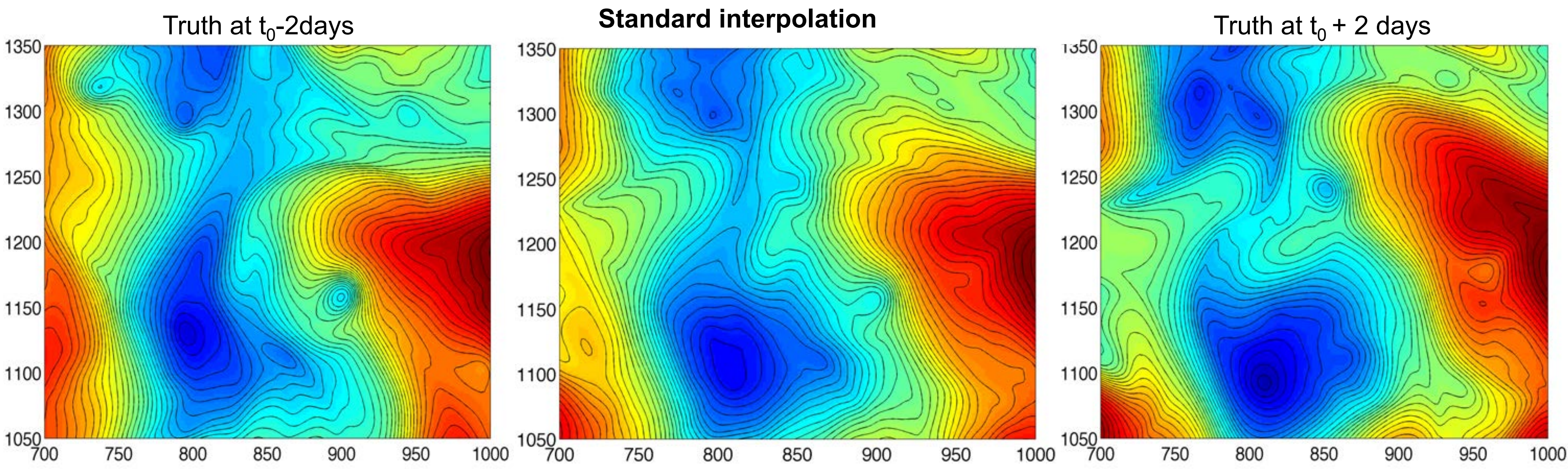

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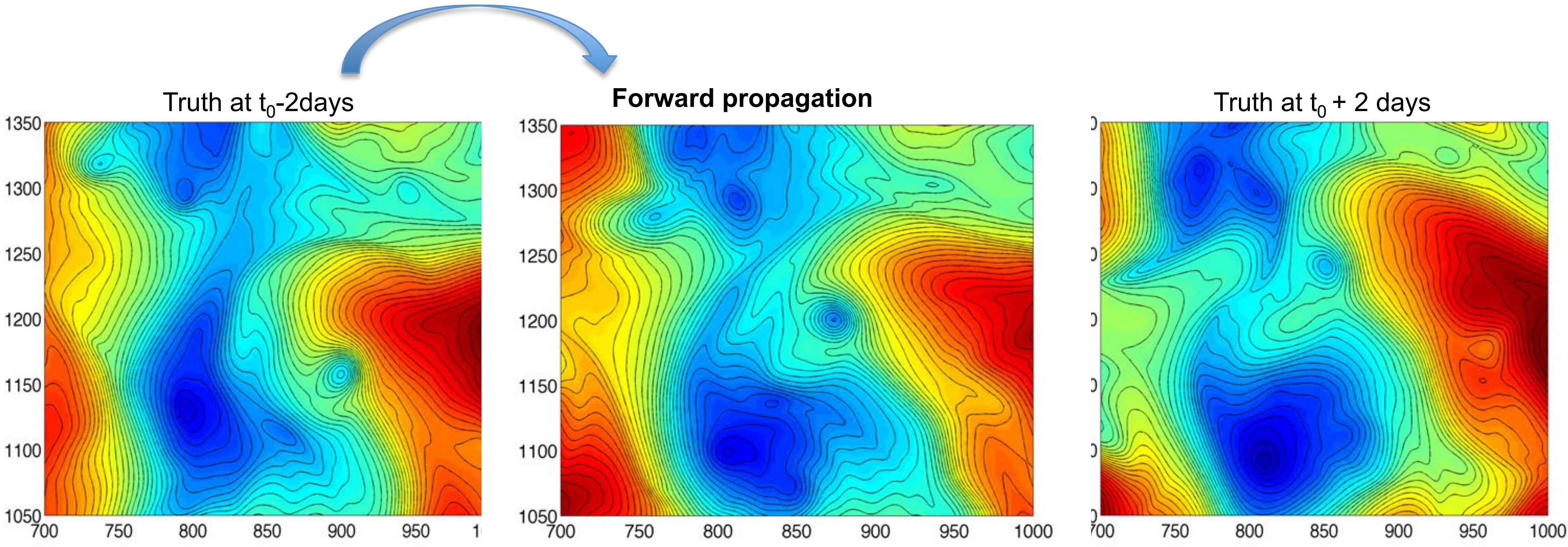

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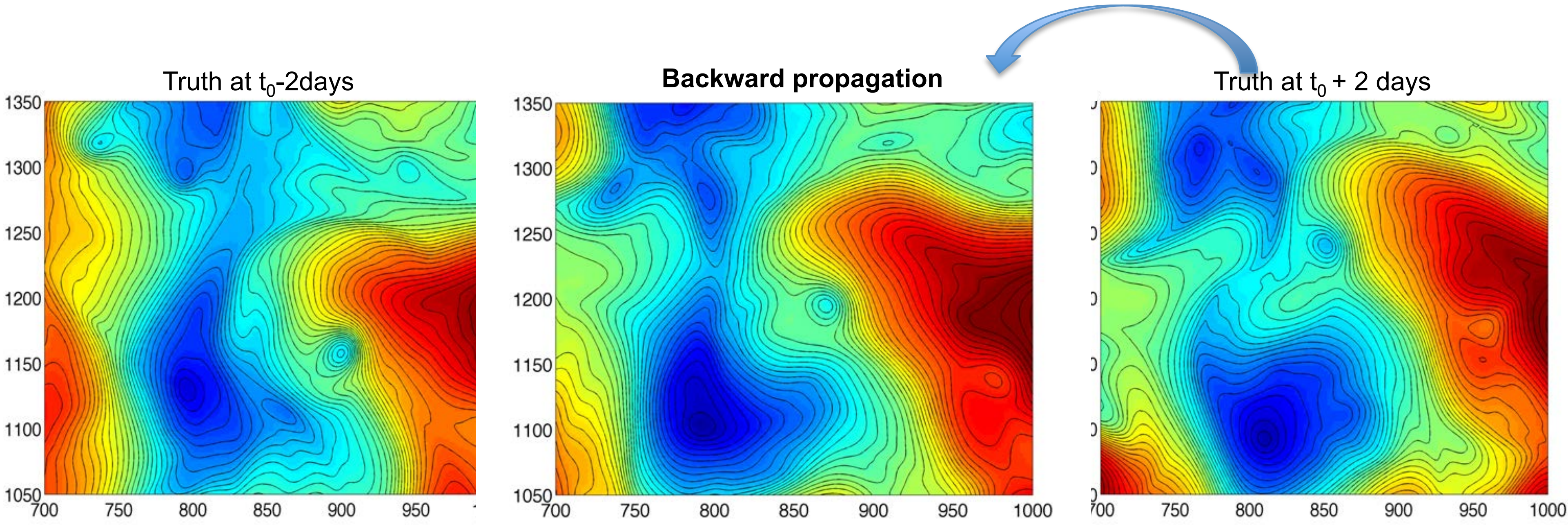

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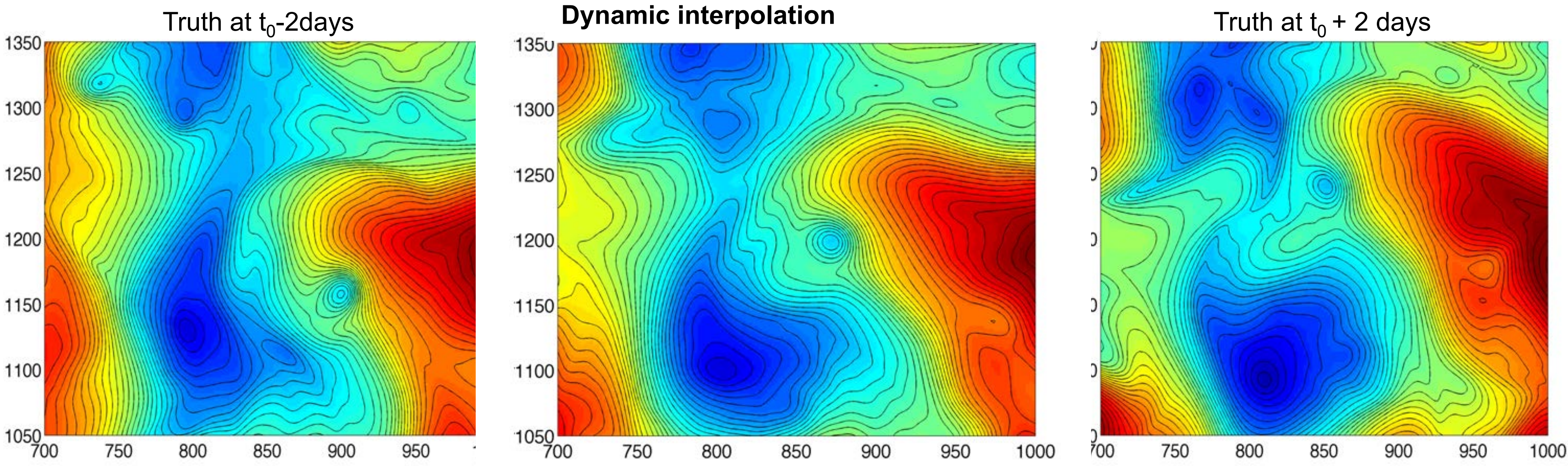

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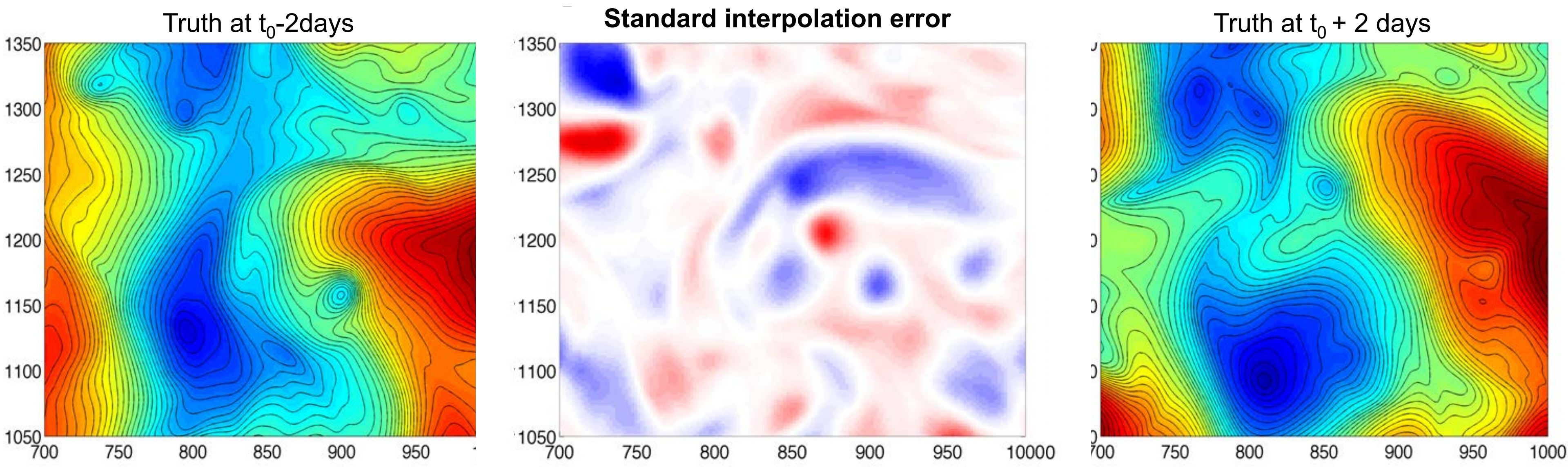

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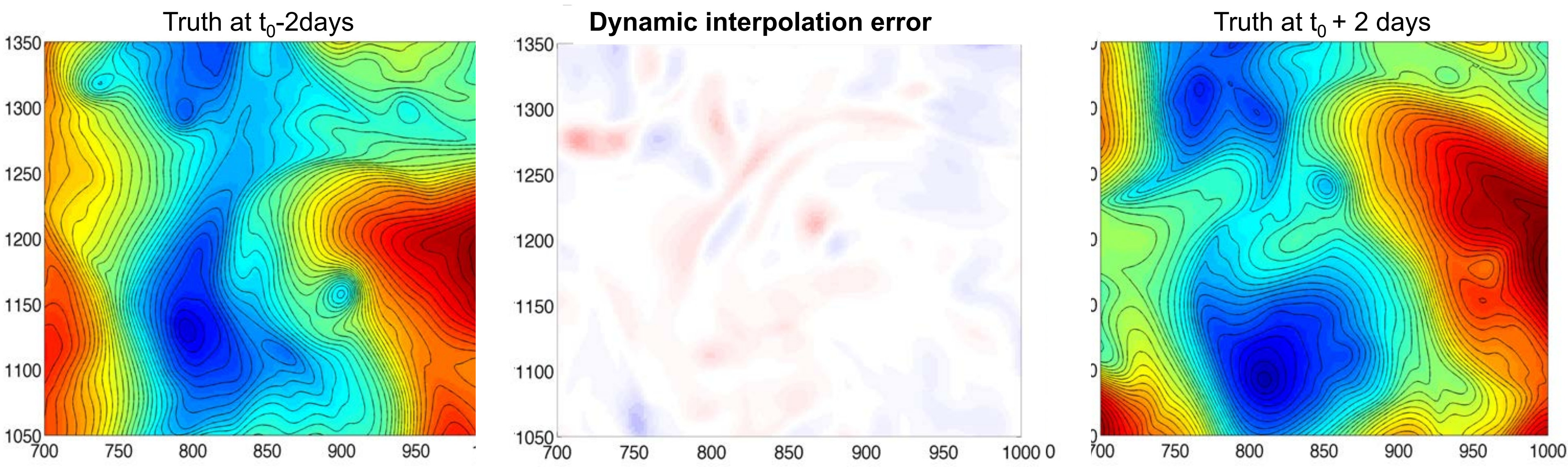

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Dynamic interpolation with realistic data: propagation of the covariances

Standard mapping with predefined covariances B

$$\mathbf{x_a} = \mathbf{x_b} + \underbrace{\mathbf{BH^t}}_{\text{Covariances between signal at tref and obs at t}}\underbrace{(\mathbf{HBH^t} + \mathbf{R})^{-1}}_{\text{Same, between pair of obs at two different times}}\underbrace{\left[\mathbf{y_o} - \mathbf{Hx_b}\right]}_{\text{Innovation with background}}$$

Covariance model:

$$B(r,t) = \left\langle SSH^2 \right\rangle \left(1 + r + \frac{1}{6}r^2 - \frac{1}{6}r^3\right) * e^{-r} e^{-\left(\frac{t}{t_0}\right)^2}$$

e.g. Arhan et Colin de Verdière, 1985

Because of the rapid mesoscale motions, covariances decrease rapidely in time (t_o~10-15 days)

→ We keep the inversion approach, but use the propagator to update covariances (B→B', with Greens function approach).
The propagator $\mathcal{M}$ is NON-LINEAR, it is linearized (**M**) around a guess → Covariances are flow-dependent

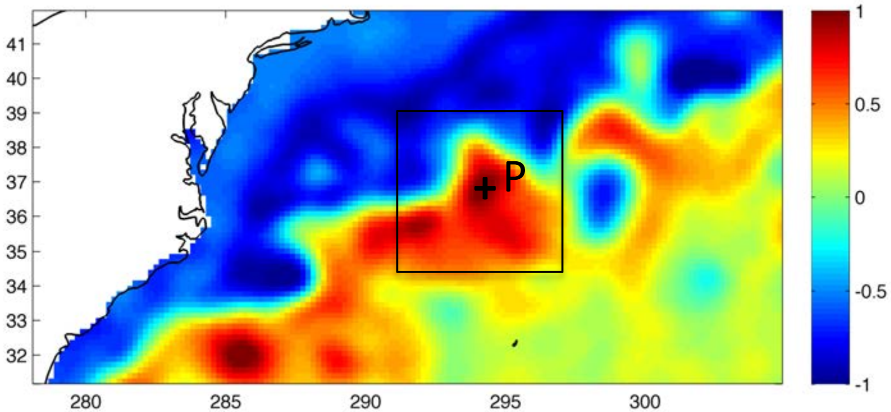
Dynamic mapping with flow-dependant covariances B'

Iterative solving (2-3 iterations enough) on the guess $\mathbf{x_g}$

$$\mathbf{x_a} = \mathbf{x_b} + \underbrace{\mathbf{B'_{x_g}}\mathbf{H^t}}_{\text{Covariances between signal at tref and obs at t transported by the linearized propagator around the guess xg}}\underbrace{(\mathbf{HB'_{x_g}H^t} + \mathbf{R})^{-1}}_{\text{Same, between pair of obs at two different times}}\left[\underbrace{\mathbf{y_o} - \mathbf{Hx_b}}_{\text{Innovation with background}} - \underbrace{\mathbf{H}\left(\mathcal{M}(\mathbf{x_g} - \mathbf{x_b}) - \mathbf{M}(\mathbf{x_g} - \mathbf{x_b})\right)}_{\text{Substraction of the non-linear evolution of the guess (not accounted in B')}}\right]$$

A part of the rapid mesoscale motions is accounted in the covariances → They are less attenuated in time, allowing better use of observations far in time

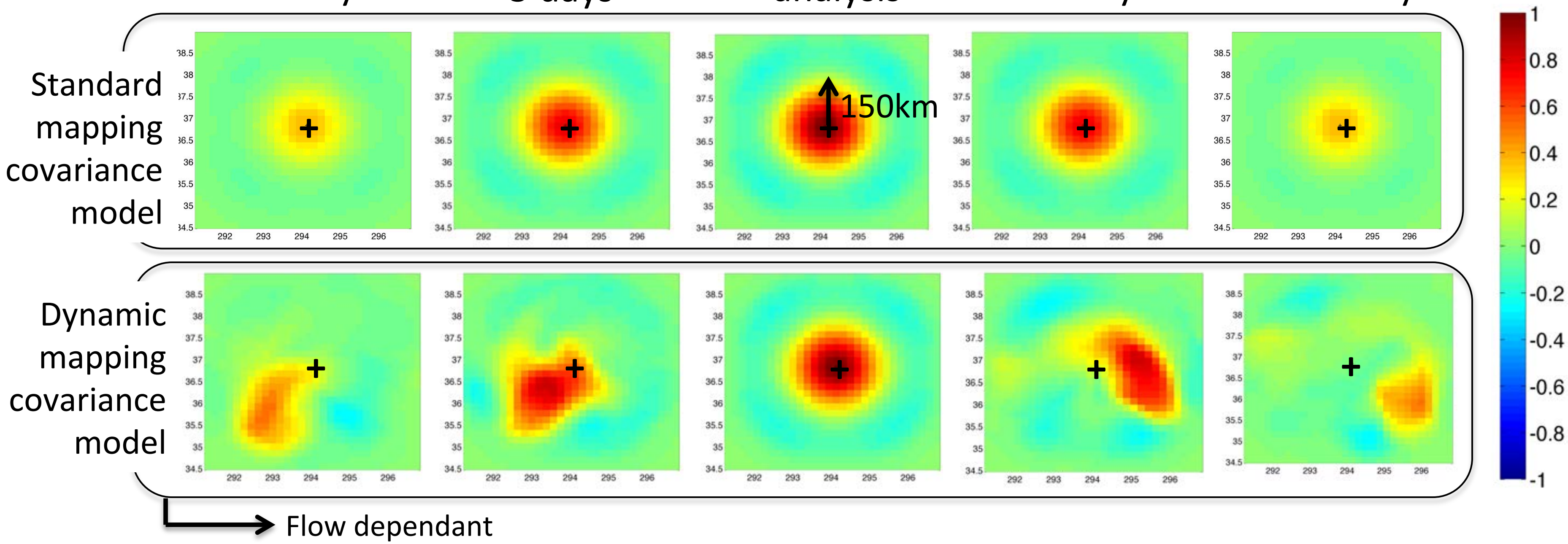
The flow-dependant covariances



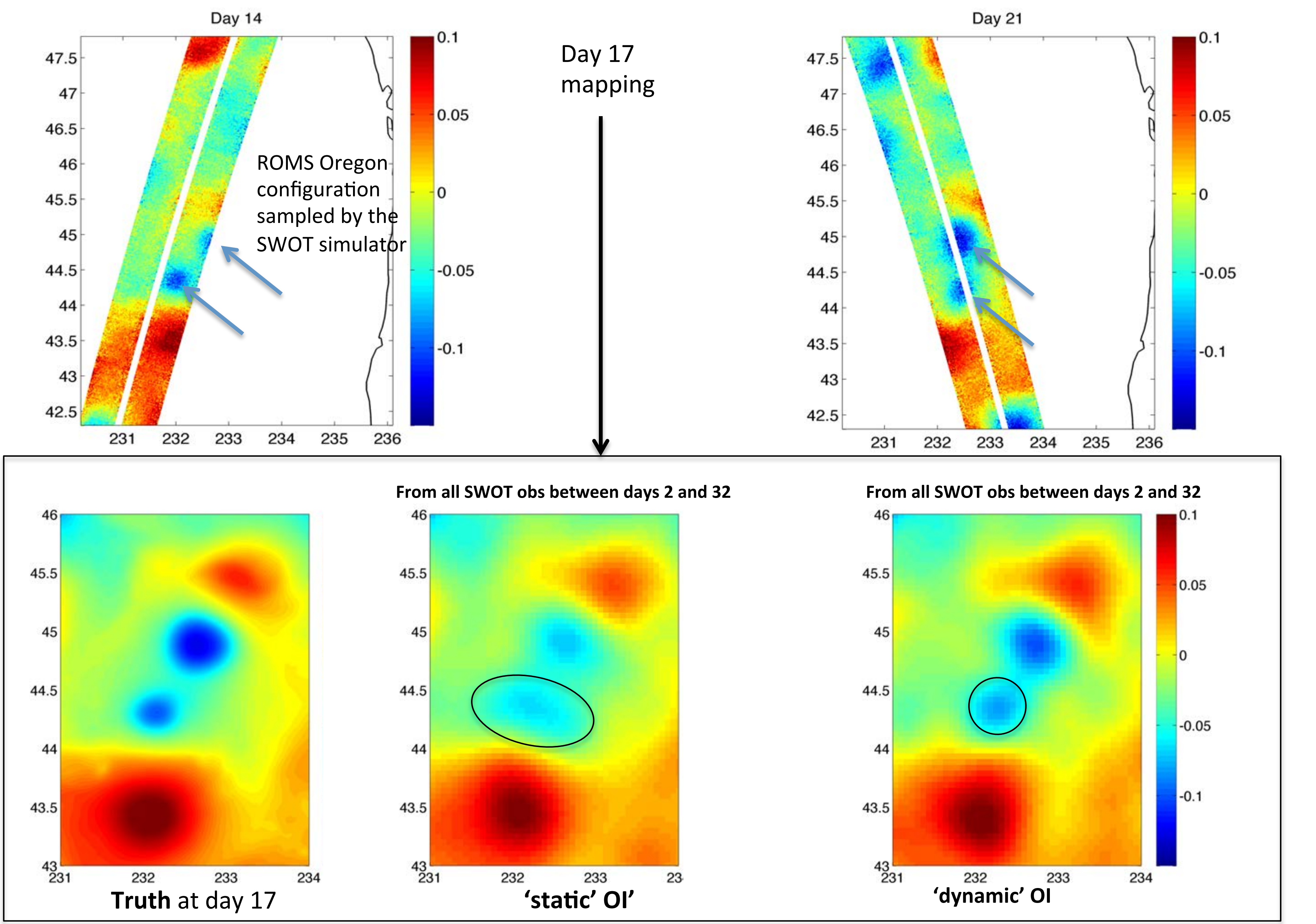
Current guess

Covariances between a grid point P at time of analysis and all grid points at any time t (representer plot):

- 10 days | - 5 days | Time of analysis | + 5 days | + 10 days

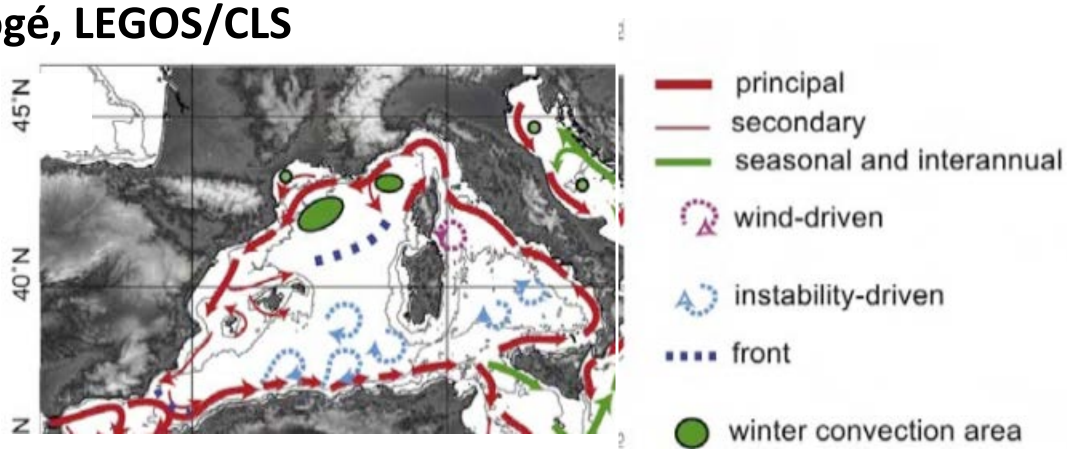


Application to SWOT synthetic data



Dealing with coastal circulations: example of Mediterranean Sea

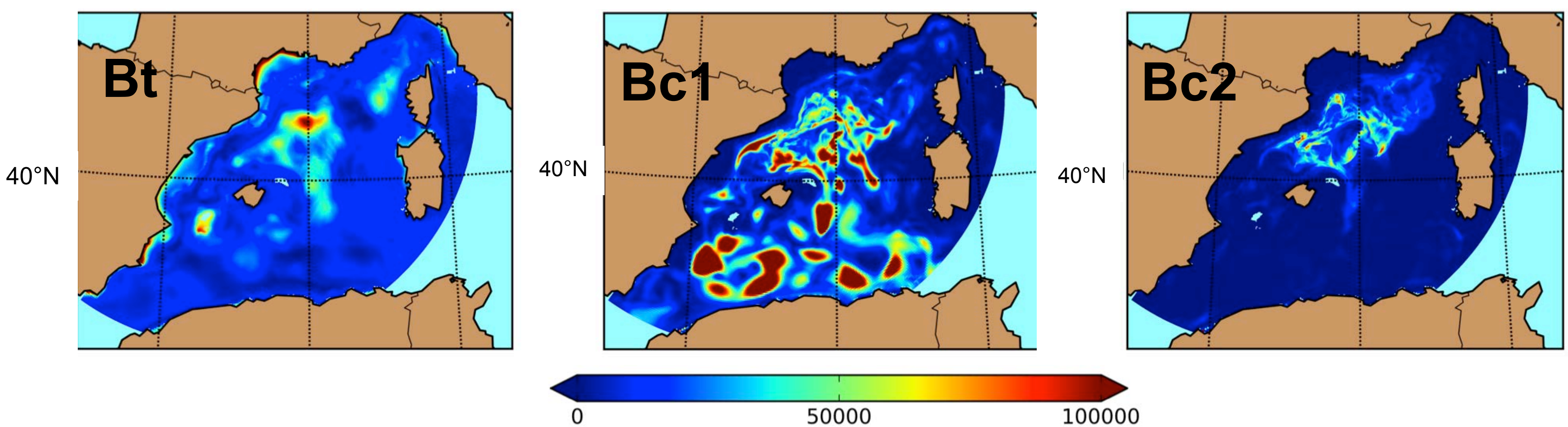
Work of M. Rogé, LEGOS/CLS



First baroclinic mode dominant

Importance of the barotropic mode near the coast

Mean variance of the pressure amplitude for barotropic and the two first baroclinic modes over Marc



Dealing with coastal circulations: example of Mediterranean Sea

Reynolds decomposition to divide quantities into a time mean plus fluctuations

PV fluctuation equation (Arbic, 2000) : $\frac{\partial q}{\partial t} + J(\psi, \overline{q}) + J(\overline{\psi}, q) + J(\psi, q)$ = *ssd*

2 layers model : adding the barotropic mode

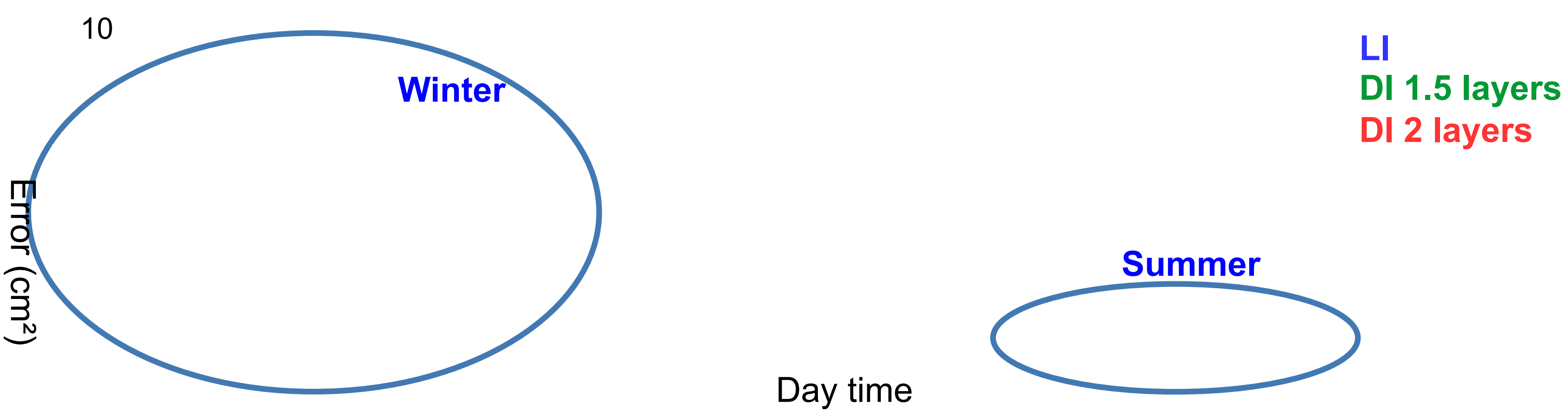
$$\psi_{bc} = \frac{\sqrt{\delta}}{1+\delta}(\psi_1 + \psi_2)$$

$$\psi_{bt} = \frac{\delta\psi_1 - \psi_2}{1+\delta}$$

$$q_{1'} = \nabla^2\psi_1 + \frac{(\psi_2 - \psi_1)}{(1+\delta)L_d^2}$$

$$q_{2'} = \nabla^2\psi_2 + \frac{\delta(\psi_1 - \psi_2)}{(1+\delta)L_d^2}$$

$$\delta = \frac{H_1}{H_2}$$



Conclusions

- We focused on the balanced dynamics (no internal waves)

- Standard objective mapping method would work for SWOT but would not handle scales below 80-100km

-The use of reduced models to compute flow-dependant covariances (dynamic mapping) is a possible approach to map smaller scales

--> 1 QG layer is already efficient in open ocean

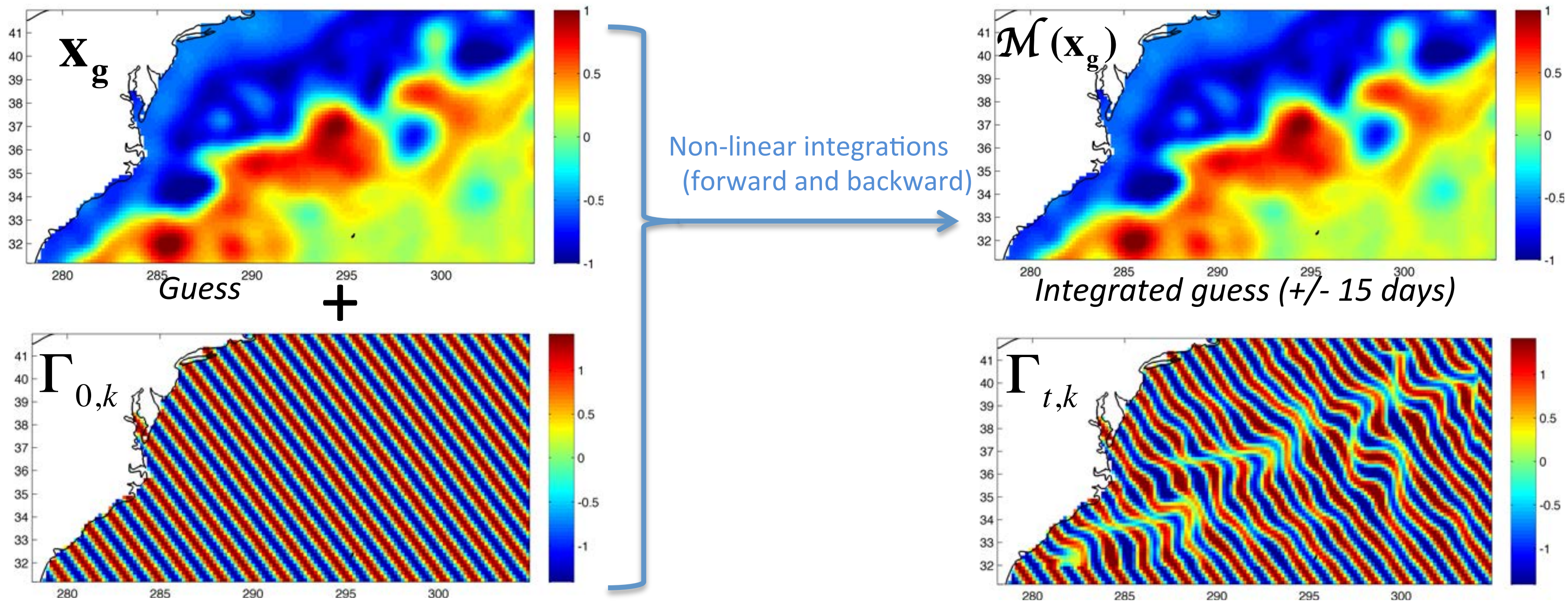
--> Possibility to consider 2 modes

- The first tests on synthetic SWOT data are promising (regional tests so far, computationally demanding)

- Should be compared with data assimilation in full PE models

backup

Computing the covariances B': the Green Function approach



Guess

Integrated guess (+/- 15 days)

One of the ~1000 Fourier component

Linear response around the guess after 2 days

$$\mathbf{G}_k = \mathbf{H}\Gamma_k$$

Γ_k represents the propagated Fourier components (or 'Green functions') by the linear response of the propagator around the guess. G (projected in obs space) is the green function matrix

$$\mathbf{B}' = \Gamma \mathbf{Q} \Gamma^{\mathrm{T}}$$

Q is diagonal, constructed consistently with the spatial covariance

Implementing the dynamic propagator in the covariance model

$$\mathbf{x_a} = \mathbf{x_b} + \underbrace{\mathbf{B'H^t}}_{\text{Covariances between signal at tref and t transported by the linearized propagator around guess xg}} \underbrace{(\mathbf{HB'H^t} + \mathbf{R})^{-1}}_{\text{Same, between pair of obs at two different times}} \left(\underbrace{\mathbf{y_o} - \mathbf{H}\mathcal{M}(\mathbf{x_g})}_{\text{Innovation with respect to the propagated guess}} + \underbrace{\mathbf{HM}(\mathbf{x_g} - \mathbf{x}_b)}_{\text{Addition of the guess innovation with respect to the background}} \right)$$

$\mathcal{M}$: the non-linear propagator
M: Linearized propagator
xg: the current guess used for the propagator’s linearization. Iterative solving on xa

B’ obtained with green functions propagated through the 1st BM PV conservation on a current guess xg.

→ A new covariance model for altimetry mapping, in the linearized 1st baroclinic mode space (instead of static or ‘translating’ space)

DOI: analysis step and iterative solving

$$\mathbf{x_a} = \mathbf{x_b} + \underbrace{\Gamma_0 \mathbf{Q}\mathbf{G}^t}_{\substack{\text{Covariances between signal at}\\ \text{tref and t transported by the}\\ \text{1stBM linearized around guess xg}}} \underbrace{(\mathbf{G}\mathbf{Q}\mathbf{G}^t + \mathbf{R})^{-1}}_{\substack{\text{Same, between}\\ \text{pair of obs at two}\\ \text{different times}}} \Big(\underbrace{\mathbf{y_o} - \mathbf{H}\mathcal{M}(\mathbf{x_g})}_{\substack{\text{Innovation with}\\ \text{respect to the}\\ \text{propagated guess}}} + \underbrace{\mathbf{H}\mathbf{M}(\mathbf{x_g} - \mathbf{x}_b)}_{\substack{\text{Substraction of the}\\ \text{non-linear part of the}\\ \text{guess integration}}} \Big)$$

xg: a current guess used for 1stBM linearization

G: sin function (2D Fourier component) propagated by 1stBM applied to xg, in obs space ('Green functions', details later)

Practical implementation

Formulation with inversion in reduced grid space :

$$\mathbf{x_a} = \mathbf{x_b} + \Gamma_0 \underbrace{(\mathbf{Q}^{-1} + \mathbf{G}^T\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{R}^{-1}\Big(\mathbf{y_o} - \mathbf{H}((\mathcal{M}(\mathbf{x_g}) - \mathbf{M}(\mathbf{x_g} - \mathbf{x}_b))\Big)}_{\eta}$$

$\mathbf{x_a} = \mathbf{x_b} + \Gamma_0 \eta$ Iterative solving on η $\mathbf{M}(\mathbf{x_g} - \mathbf{x}_b) = \mathbf{G}\eta_g$

$$\eta = (\mathbf{Q}^{-1} + \mathbf{G}^T\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{R}^{-1}(\mathbf{y_d} + \mathbf{G}\eta_g)$$

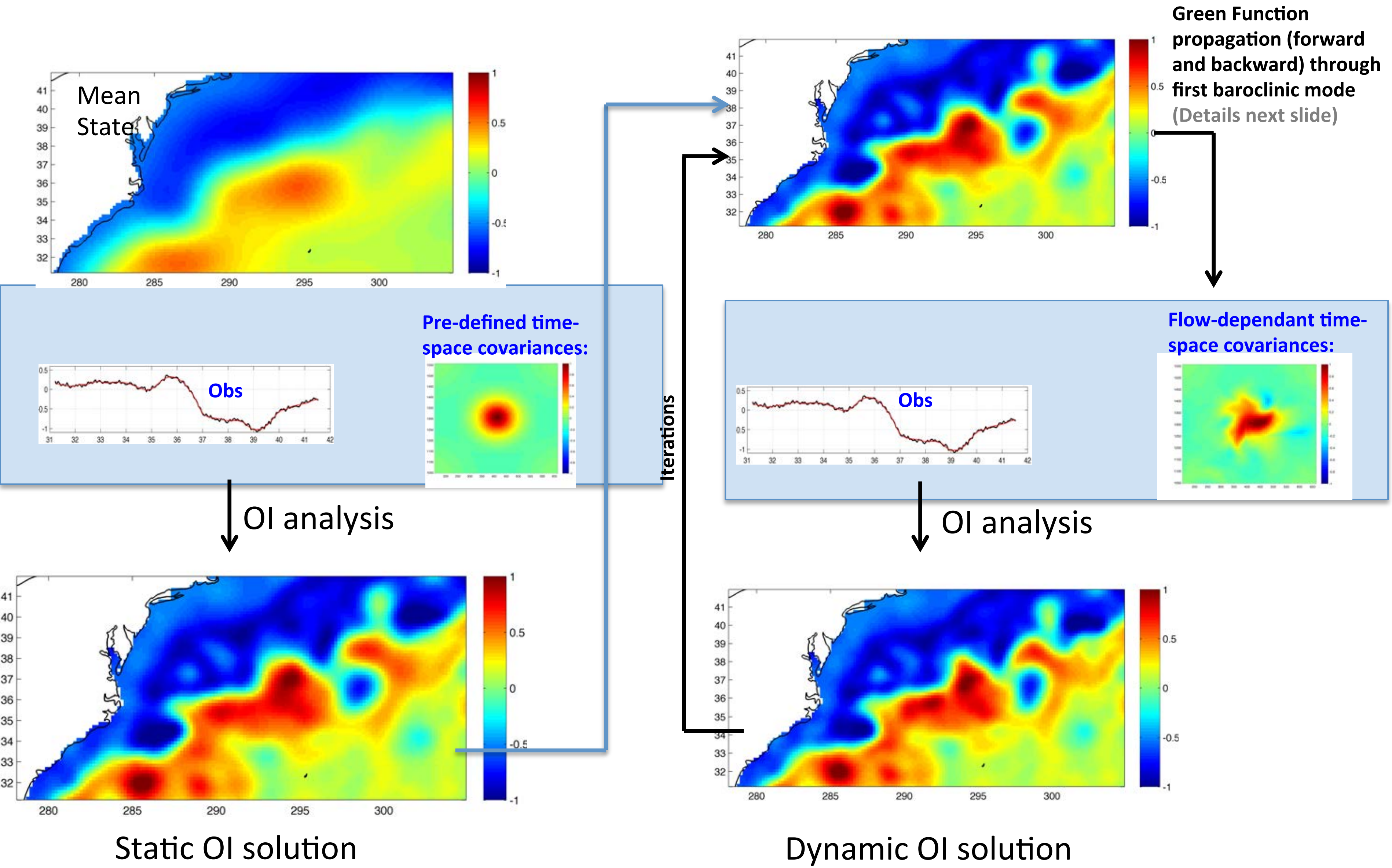
Update G with M forward and backward integrations

Update η

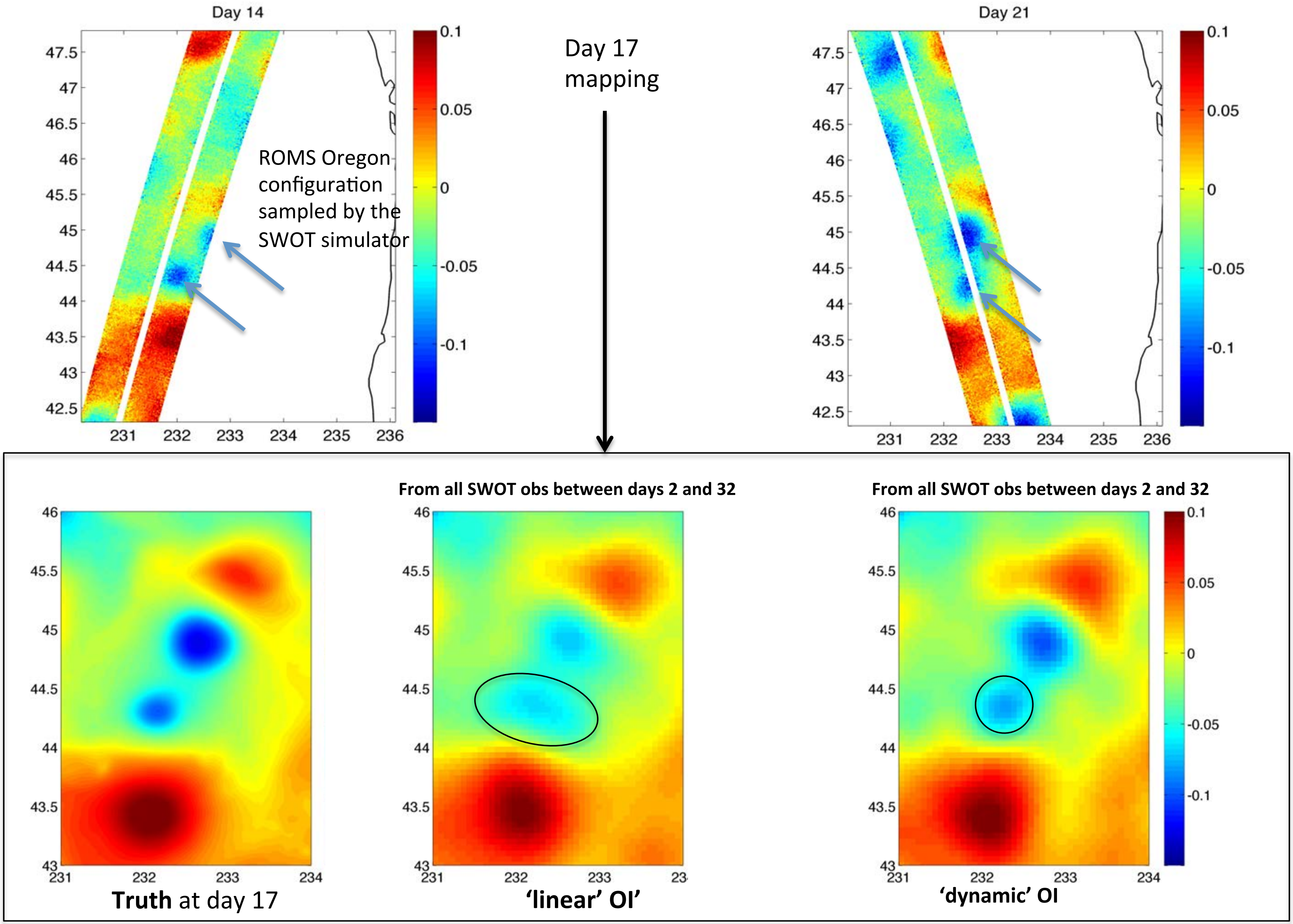
A local analysis is performed (not detailed here) → allows to limit the size of the Fourier decomposition (<600km) regardless the size of the domain → ~200 components

DOI solution: $\mathbf{x_a} = \mathbf{x}_b + \Gamma_0 \eta$

Illustration of iterative solving



In the context of SWOT



How distinct from data assimilation in OGCM ?

- This dynamic OI uses data assimilation techniques, but it is disctinct from data assimilation in OGCM:

→ **DA in OGCM** is generally under-observed (e.g. N vertical modes, but just SSH is observed…) → **the model fill the unknowns with its own physics.**

→ **With this dynamic OI, the strong constraint is on the data:** the system is not under-observed (only 1 baroclinic mode accounted in the model) and the N-1 remaining modes are parameterized with tapered covariances in time.
Just as the N modes are pamameterized for the standard OI.

Also, the 'restoring force' is toward a mean state, not an OGCM state.