Inference of the bedrock topography beneath glaciers in interior sectors of East Antarctica by multi-sensor data assimilation

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Objective

This research project aims at improving the estimation of the bed topography beneath ice-sheets where no airborne measurements are available. The knowledge of the bed topography is a basic step in setting up numerical flow models. Moreover, combined with the surface measurements, this provides volume estimations.
The current estimations are derived from multiple airborne ice thickness surveys and Kriging interpolation techniques, see [Bamber et al. 2013] for Greenland and [Fretwell et al. 2013] for Antarctica (BedMap2 project, international dataset, map produced by the British Antarctic Survey). In presence of relatively dense airborne measurements, e.g. CReSIS radar datasets, the bed topography elevation beneath fast sliding ice-streams (~100 m/y and more) can be estimated by inverting a mass conservation equation, see the BedMachine project [Morlighem et al. 2017]. However, inferring the bed topography elevation in regions where very few airborne measurements are available, moreover where the glaciers are partially slipping - partially sheared, remains a scientific challenge, see e.g. [ModeBo17] and references therein.

In East Antarctica Ice Sheet (EAIS) areas that are more than 50 km from direct ice-thickness measurements the estimations uncertainties are large, up to +/- 1000m [Fretwell et al. 2013]. The objective of the present project is to improve these bed topography estimations in interior sectors of EAIS where the current estimations are still uncertain.

Method

Given satellites measurements of the ice surface (elevation and velocity), the difficulty is to separate the effects of the bed shape from the glacier basal sliding and from the varying vertical ice rate factor profile (thermal vertical gradient), [ModeBo17]. To solve this "signature separation problem", an original Reduced Uncertainty (RU) formulation of the non-isothermal Shallow Ice Approximation (SIA) model has been derived. This is the so-called RU-SIA flow model, [ModeBo17, MoZh19]. It takes into account both the mass conservation *and* the momentum conservation. The ice thickness is the key unknown parameter of the equation to estimate. The RU-SIA model is then "inverted" from the surface measurements, with the help of a Deep Neural Network (trained from airborne measurement datasets) and a Variational Data Assimilation algorithm, [MoZh21].

The resulting estimations are valid at relatively large scale, that is with minimal wavelength ~10 times the thickness (~30 km in EAIS). The remarkable property of these estimations is the following: they are theoretically valid inland where the surface ice flows are relatively slow (~10-50+ m/y), and without dense airborne measurements.

Results: demonstration of the robustness

On Fig. 1 are presented the EAIS regions (named Ant p) where the RU-SIA model is theoretically valid and where it has been inverted to estimate the bedrock elevation. Below are presented the results in a highly measured area located in the here called "Ant2" region of EAIS, see Fig 2. The obtained estimations are compared to the two reference bedrock estimations Bedmap2 [Fretwell et al. 2013] and Bedmachine [Morlighem et al., 2020]. On the contrary to these two aforementioned methods, the current method can be applied in unmonitored areas presenting low flow velocities (10-50+ m/y).

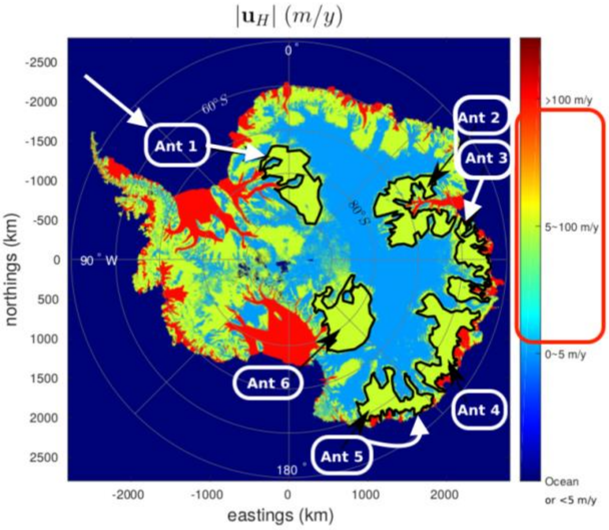

Figure 1. Antartica surface velocity (from CSA, JAXA and ESA data), [Rignot et al.], with the regions Ant-p where the inverse method based on the RU-SIA model is a-priori valid and has been applied.

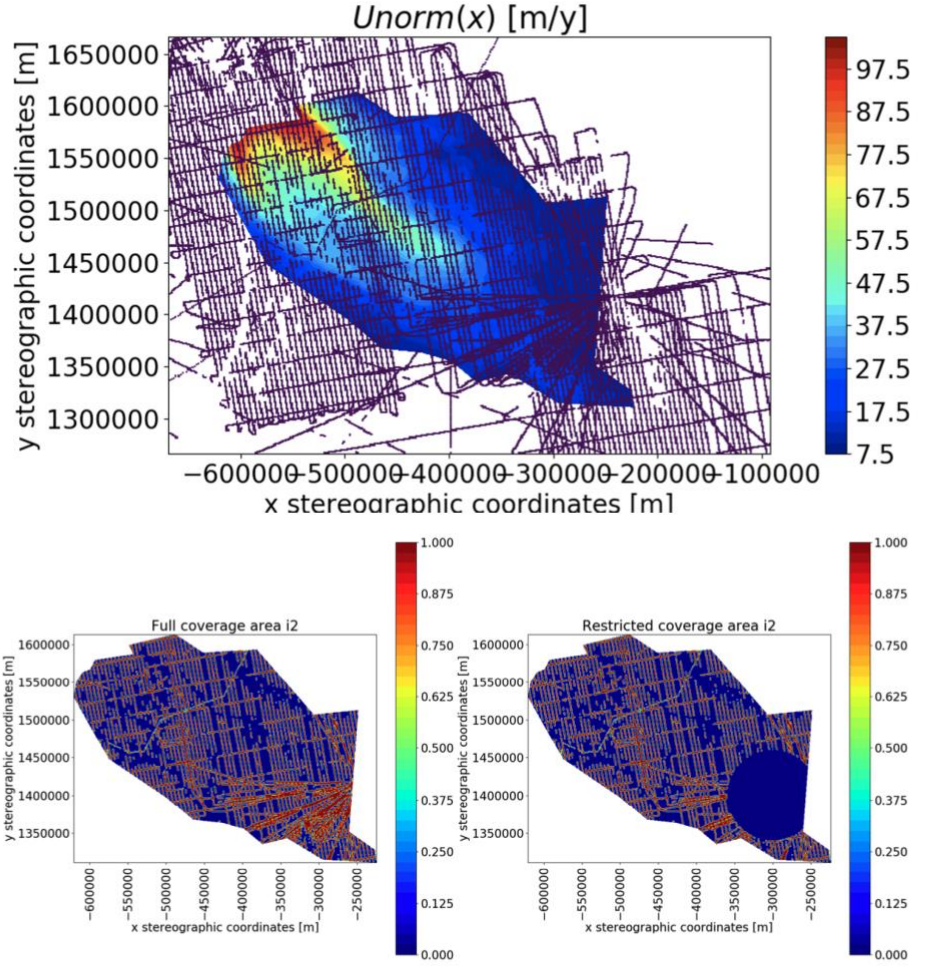

Figure 2. EAIS: zoom on a highly measured area in Ant 2 region (see Fig. 1). (Up) Surface velocity values. (Down)(L) Lines representing the tracks of the airborne ice thickness measurements. (Down)(R) Same as (L) but with the airborne measurements which have been artificially withdrawn in a 120 km diameter circle. This enables to evaluate the robustness of the present estimation method.

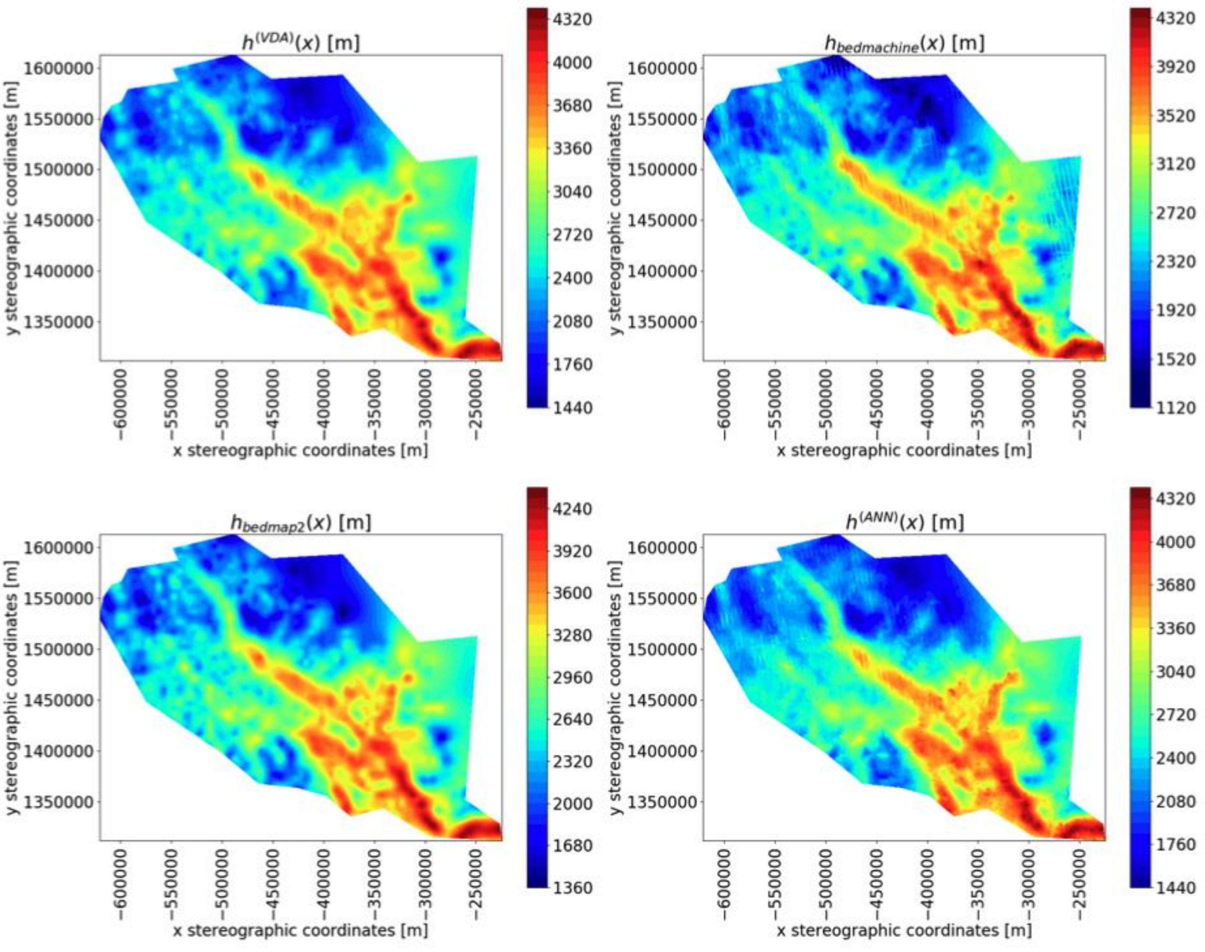

Figure 3. Ice thickness estimations in the highly measured area (Ant 2 region). (Up) (L) The present estimation named h(VDA) (m) [Moetal21] (R) BedMachine estimation h(bedmachine) [Morlighem et al. 2020].
(Down)(L) Bedmap2 estimations h(bedmap2) [Fretwell et al. 2013]. (R) A purely data driven estimation h(ANN) obtained in [Moetal21].
The colormaps are the same for all sub-figures (despite min and max values can be different).

In this densely measured region, the four estimations are similar, Fig. 3. However, it can be noticed that in low velocities areas, the estimation h(BedMachine) does not accurately fit the airborne measurements. The same remark holds for the present purely data driven estimation h(ANN), see the lines on Fig. 3 (R) (Up) and (Down).
The obtained differences between the present estimations h(VDA) and Bedmap2 h(bedmap2) equal here 11.5 m in median, 28.1 in mean and 645 m in max.

The sensitivities of the present method with respect to the surface data smoothing length scales, the ANN model, the grid size, the flight tracks density and/or locations and the first guess values, have been thoroughly investigated. These investigations (led on 8 large areas inland Antarctica and Greenland too) have demonstrated a good robustness of the inverse method, [MoZh21]. Its critical sensitivity is with respect to the accuracy (at the targeted scale) of the ice surface slope, [Moetal21].

The very interesting feature of the present method is the following. Since the method relies on a mathematically stable inverse model, the method remains relatively robust with respect to the flight tracks locations or density. To illustrate this point, the same case as above is performed but with the artificially withdrawn airborne measurements in the 120 km diameter circle indicated on Fig. 4. Moreover, in this (artificially) unmonitored area, the ice thickness greatly varies. This makes the test case very challenging.

The obtained estimations in the unmonitored area (the circle) remain fairly good, see Fig. 4. The estimation h(VDA) in this area are quite similar if considering the tracks values or not. Therefore, the differences between h(VDA) and h(bedmpa2) remain similar to those previously indicated.

Recall that the currently reference methods Bedmap2 and Bedmachine would be “blind” in this area where there is no ice thickness measurement and where the thickness greatly varies.

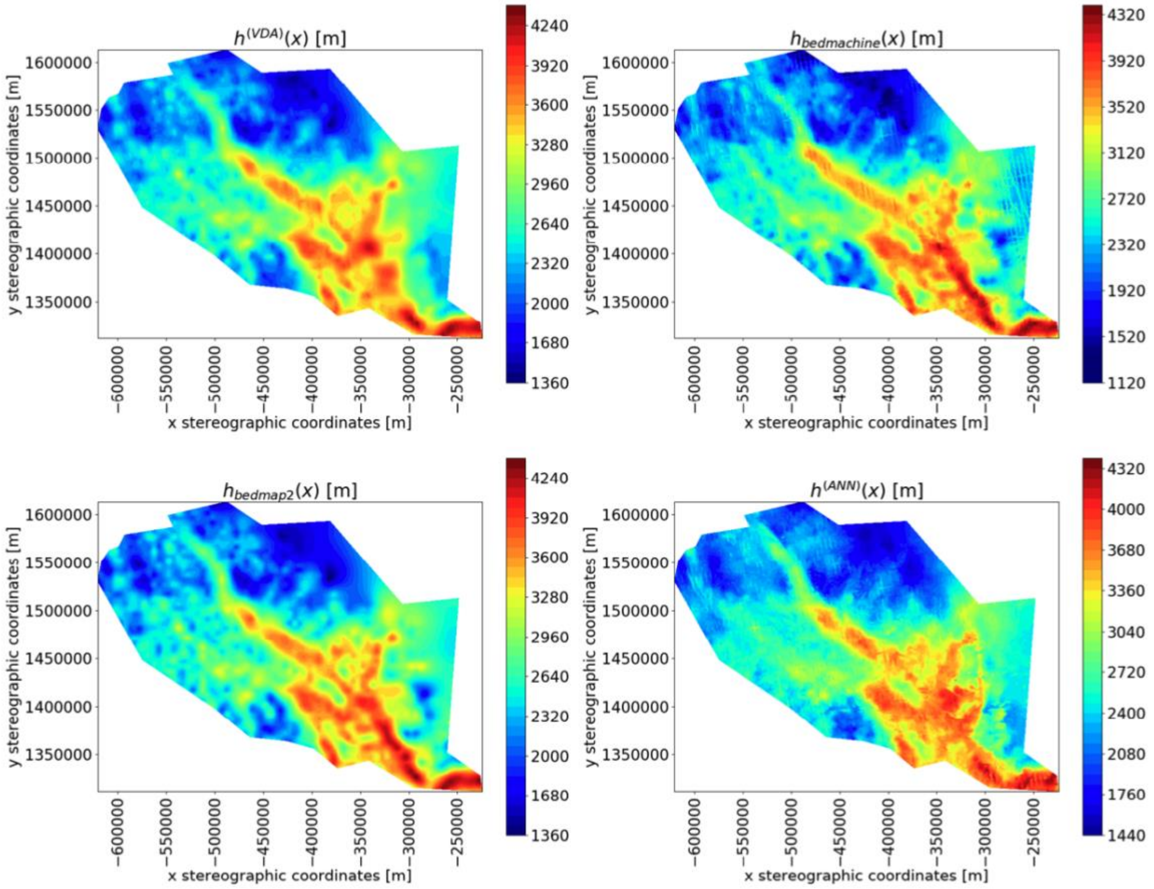

Figure 4. Same as Fig. 3 but here h(VDA) (Up)(L) and h(ANN) (Down)(R) have been obtained without the airborne measurements in the 120 km diameter circle.

The reference BedMachine estimations (Up)(R) and Bedmap2 estimations (Down)(L) remain the same as in Fig. 3.

The colormaps are the same for the four plots (despite the min and max values differ).

Finally, the developed method enables to provide fairly good estimations of bedrock elevation in regions where airborne measurements are not available, moreover in regions where the ice flows slowly (~10-50+ m/y). In terms of inference, these far inland regions represent highly challenging scientific problems. The developed hybrid Machine Learning / physically-based algorithm enables to improve the estimations of ice thickness (bedrock elevation) in these large unmeasured inland areas of EAIS.

References

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