The Wavenumber Spectra and Standard Deviations of Uncorrelated Errors in SWOT Measurements of Sea-Surface Height for Various Footprint Sizes

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Abstract. The science requirement specification of the errors in SWOT measurements of sea-surface height is expressed in the form of an along-track wavenumber spectrum after smoothing the data with a half-power filter cutoff wavelength of 15 km. This expression of the measurement errors is inconvenient for users who want to analyze SWOT data without applying 15-km smoothing. Higher-resolution SWOT data are primarily affected by the contribution of uncorrelated instrumental noise to the total measurement errors. The purpose of this note is to derive from the science requirement specification the corresponding wavenumber spectra and standard deviations of the uncorrelated measurement errors for footprint sizes of 0.5 km, 1.0 km and 2.0 km, without 15-km smoothing in the across-track dimension. The analysis assumes that the spectra of the uncorrelated errors are white at wavelengths shorter than 15 km for each of these footprint sizes.

The errors of SWOT measurements of sea surface height (SSH) can be partitioned into a contribution from uncorrelated measurement errors and a contribution from long-wavelength errors attributable to orbit height errors, antenna pointing errors, errors in biases from significant wave height effects, and errors in the corrections for various geophysical effects on the range estimates. As specified in the SWOT documentation (Rodríguez and Callahan, 2018; Peral, 2016; Esteban Fernandez, 2017), the science requirement for the across-track average of the along-track wavenumber spectrum of the long-wavelength errors is

$$S_{red}(l) = 0.0125\ l^{-2}\ \text{cm}^2/\text{cpkm}, \qquad \text{for } 1/1000 < l < 1/15 \text{ cpkm}, \tag{1}$$

where l is the along-track wavenumber and cpkm is shorthand for cycles per km. For reasons discussed below, the SWOT documentation does not specify this "red noise" spectrum for wavenumbers higher than $l = 1/15$ cpkm (i.e., wavelengths shorter than $\lambda = l^{-1} = 15$ km). The spectrum (1) of long-wavelength errors is shown extrapolated beyond $l = 1/15$ cpkm by the red dotted line in Fig. 1.

The details of the spectrum of uncorrelated measurement errors depend on the smoothing applied to the SWOT data, either in the onboard pre-processing or in ground-based post-processing. The original plan for the onboard pre-processing of SWOT data over the oceans was to smooth the raw SWOT measurements of SSH to achieve a footprint size of 1 km on a 1 km × 1 km grid. The along-track wavenumber spectrum of these uncorrelated measurement errors is assumed to be "white" (constant for all wavenumbers), as shown by the blue dashed line in Fig. 1 that was deduced from the science requirements as summarized below. The green and purple dashed lines are the white noise spectra for alternative footprint sizes of 0.5 km and 2 km, respectively, which are discussed later in this document.

The goal of the SWOT mission is to resolve SSH on wavelength scales longer than 15 km over 68% of the world ocean (Rodríguez and Callahan, 2018). The science requirement for the uncorrelated measurement errors is therefore specified in the SWOT documentation (Rodríguez and Callahan, 2018; Peral, 2016; and Esteban Fernandez, 2017) in terms of the residual errors after smoothing in ground-based post-processing with a half-power filter cutoff wavelength of $\lambda_c = 15$ km,

$$\overline{S}_{1\mathrm{km}}(l) = 2\ \mathrm{cm}^2/\mathrm{cpkm}, \qquad \text{for } 1/1000 \le l \le 1/15\ \mathrm{cpkm}. \tag{2}$$

The overbar signifies the 15-km smoothing of the SWOT data and the subscript 1km signifies a footprint size of 1 km. This residual white-noise spectrum after 15-km smoothing of the 1 km $\times$ 1 km pre-processed SWOT data is shown by the red dashed line in Fig. 1. As in the case of the spectrum (1) of long-wavelength errors, the SWOT documentation does not specify the spectrum (2) of 15-km smoothed uncorrelated measurement errors for wavenumbers higher than $l_c = \lambda_c^{-1} = 1/15$ cpkm. If the 15-km smoothing is applied 1-dimensionally in the across-track dimension, the along-track spectrum of $\overline{S}_{1\mathrm{km}}(l)$ would have the value (2) for all wavenumbers l up to the Nyquist wavenumber $l_{\mathcal{N}} = (2\Delta y)^{-1}$, which is 0.5 cpkm for an along-track grid spacing of $\Delta y = 1$ km and a footprint size of 1 km. If the 15-km smoothing is applied 2-dimensionally, the along-track spectrum (2) would be attenuated at the wavenumbers higher than l_c.

It is evident from the spectrum shown by the red dotted line in Fig. 1 that the long-wavelength errors have very little power at wavenumbers higher than the filter cutoff wavenumber $l_c = 1/15$ cpkm. Smoothing with a half-power filter cutoff wavelength of $\lambda_c = 15$ km would thus have little effect on the long-wavelength errors. The spectrum of the total measurement errors in 15-km smoothed SWOT data would therefore consist essentially of the sum of the contributions (1) and (2), which is shown by the solid red line in Fig. 1. The total error spectrum at wavenumbers $l > l_c$ would continue to asymptote toward an extended red dashed line if the 15-km smoothing is applied 1-dimensionally in the across-track dimension and the spectral characteristics of the uncorrelated errors are white at wavenumbers higher than l_c. This spectrum of the total errors intersects the 68$^{\mathrm{th}}$ percentile of the global SSH spectrum (shown by the solid black line in Fig. 1) at a wavelength of about 15 km, thus achieving a signal-to-noise ratio of approximately 1 at the 15-km wavelength that is the resolution goal of the SWOT mission.

The science requirement for the uncorrelated measurement errors in the onboard pre-processed SWOT estimates of SSH is thus specified in the SWOT documentation in the rather inconvenient form (2) in terms of the spectral characteristics of the errors after applying 15-km smoothing in ground-based post-processing. For many applications, it is desirable to know the characteristics of the uncorrelated measurement errors in the pre-processed data without 15-km smoothing in ground-based post-processing. This can be determined straightforwardly from (2) as follows (see Appendix F of Chelton et al., 2019, for more details).

For estimation of the 1-dimensional along-track wavenumber spectrum of the noisy onboard pre-processed SWOT measurements of SSH, across-track smoothing with a half-power filter cutoff wavelength of λ_c attenuates the along-track white-noise spectrum of uncorrelated measurement errors by the factor $\nu = 2\Delta x \lambda_c^{-1}$ [see Eqn (E.11b) in Appendix E of Chelton et al. (2019)], where Δx is the across-track grid spacing between uncorrelated measurement errors, which is equal to the measurement footprint diameter. If smoothing of the onboard pre-processed SWOT data is applied two-dimensionally and isotropically with an ideal (but unrealizable) filter, the along-track smoothing would additionally attenuate the spectrum to zero at along-track wavenumbers between $l_c = 1/15$ cpkm and the Nyquist wavenumber $l_{\mathcal{N}} = (2\Delta y)^{-1}$. For a realizable 2-dimensional filter, the spectrum of residual errors after smoothing will exhibit the imperfections of a gradual rolloff through the half-power filter cutoff wavenumber l_c, along with the higher-wavenumber sidelobes of the realizable filter. These imperfections of the filter transfer function are particular to the choice of filter used to smooth the onboard pre-processed SWOT data. The result for 2-dimensional filtering with a Parzen smoother are shown by the green lines in Fig. F.2 of Chelton et al. (2019).

For a footprint size of 1 km, the grid spacing between uncorrelated measurement errors is $\Delta x = \Delta y = 1$ km. With the filter cutoff wavelength of $\lambda_c = 15$ km considered above based on the science requirement specifications for SWOT, the attenuation factor from the across-track smoothing would then be $\nu = 1/7.5$. The white-noise spectrum of uncorrelated SWOT measurement errors extrapolated from the science requirement (2) for the case of a footprint size of 1 km without 15-km smoothing would therefore be a factor of 7.5 times larger than the white-noise spectrum (2) of the 15-km smoothed SWOT data,

$$S_{1\mathrm{km}}(l) = \frac{1}{\nu}\,\overline{S}_{1\mathrm{km}}(l) = 15\ \mathrm{cm}^2/\mathrm{cpkm}. \tag{3}$$

This white-noise spectrum of the unsmoothed onboard pre-processed SWOT data is shown by the blue dashed line in Fig. 1. Assuming that the uncorrelated measurement errors are white at wavenumbers higher than $l_c = 1/15$ cpkm, the in-orbit performance of SWOT can be easily validated from the spectrum (3) of pre-processed SWOT data with 1-km footprint size, rather than introducing the unnecessary complexity of validating the performance from the spectrum (2) of SSH after 15-km smoothing in ground-based post-processing.

The variance of the uncorrelated measurement errors in the onboard pre-processed SWOT estimates of SSH without smoothing in ground-based post-processing can be obtained by Parseval's Theorem as the integral of the constant white-noise spectrum (3),

$$\sigma^2_{1\mathrm{km}} = \int_0^{l_{\mathcal{N}}} S_{1\mathrm{km}}(l)\,dl = l_{\mathcal{N}}\,S_{1\mathrm{km}}(l) = 7.5\ \mathrm{cm}^2. \tag{4}$$

This expression for $\sigma^2_{1\mathrm{km}}$ is based on the Nyquist wavenumber $l_{\mathcal{N}} = (2\Delta y)^{-1}$ for a footprint size of 1 km and a grid spacing of $\Delta y = 1$ km in the onboard pre-processed SWOT data. The corresponding standard deviation of the uncorrelated measurement errors in the onboard pre-processed SWOT data with 1 km $\times$ 1 km grid spacing and a footprint size of 1 km is thus

$$\sigma_{1\mathrm{km}} = 2.74\ \mathrm{cm}. \tag{5}$$

It can be noted that slightly smaller values of 2.4 cm, 2.5 cm and 2.54 cm for $\sigma_{1\text{km}}$ are stated without explanation in Peral (2016) and Esteban Fernandez (2017). These lower values may be the projected best estimates of what will actually be achieved on orbit. Or they may consider only the effects of instrumental errors and neglect smaller sources of non-instrumental uncorrelated errors such as the effects of spacecraft pointing errors on the signal-to-noise ratio across the measurement swaths.

The above analysis is for the overall average standard deviation of uncorrelated measurement errors across the SWOT measurement swaths for a footprint size of 1 km and a significant wave height (SWH) of 2 m. In actuality, the errors are smallest near the middle of each measurement swath and increase toward both edges of the swaths. They also increase monotonically with increasing SWH. The projected estimates of the dependencies of the standard deviation of uncorrelated measurement errors on SWH and swath location are shown in Fig. 2. The corresponding across-track variation of the white-noise along-track wavenumber spectral value of the uncorrelated measurement errors can be determined by inverting (4) to obtain $S_{1\text{km}}(l) = l_{\mathcal{N}}^{-1}\,\sigma_{1\text{km}}^2$ based on the measurement error variance $\sigma_{1\text{km}}^2$ in Fig. 2 for the SWH and swath location of interest.

The large across-swath average standard deviation (5) for the onboard pre-processed SWOT estimates of SSH with a footprint size of 1 km can be reduced by spatial smoothing in ground-based post-processing. The results are shown in Fig. 3 for the case of isotropic smoothing with the half-power filter cutoff wavelengths λ_c shown along the abscissa. The error standard deviation decreases rapidly with increased smoothing, decaying as approximately λ_c^{-1}.

The original plan for onboard pre-processing of the SWOT data over the oceans to achieve the footprint size of 1 km that was considered above has evolved to a new plan to pre-process the data onboard the satellite to achieve a footprint size of 0.5 km. The errors of SSH estimates with this smaller footprint size are uncorrelated on a sample grid of 0.5 km × 0.5 km, but the recommendation is to post the SSH estimates on an oversampled 0.25 km × 0.25 km grid. The variance of the errors for the 0.5-km footprint diameter is unchanged on the oversampled grid but the errors at neighboring 0.25 km × 0.25 km grid points are correlated. Since there are four measurements with 0.5-km footprint in each 1 km × 1 km area, the variance of the uncorrelated errors with a footprint size of 0.5 km is a factor-of-4 larger than (4),

$$\sigma_{0.5\text{km}}^2 = 4\,\sigma_{1\text{km}}^2 = 30\ \text{cm}^2. \tag{6}$$

The standard deviation of the uncorrelated errors with a footprint size of 0.5 km is therefore

$$\sigma_{0.5\text{km}} = 5.48\ \text{cm}. \tag{7}$$

The oversampling on a 0.25 km × 0.25 km grid does not alter the wavenumber spectrum of the noise at wavenumbers smaller than the Nyquist wavenumber of 1 cpkm associated with the 0.5 km × 0.5 km grid spacing for uncorrelated errors. The constant white noise spectrum of these uncorrelated errors for the higher Nyquist wavenumber of $l_{\mathcal{N}} = 1$ cpkm associated with the grid spacing of $\Delta y = 0.5$ km can be obtained from (6) by inverting an expression analogous to (4) to get

$$S_{0.5\text{km}}(l) = \frac{1}{l_{\mathcal{N}}}\,\sigma_{0.5\text{km}}^2 = 30\ \text{cm}^2/\text{cpkm}. \tag{8}$$

This spectrum is shown by the green dashed line in Fig. 1. Note that the statistics (6), (7) and (8) for the uncorrelated errors in SWOT estimates of SSH with a footprint diameter of 0.5 km are not science requirements for SWOT. The only science requirement for uncorrelated measurement errors is the spectrum (2) obtained after smoothing the onboard pre-processed SWOT data 1-dimensionally with a half-power filter cutoff wavelength of 15 km in the across-track dimension.

Because of the large uncorrelated measurement errors in SSH estimates with a footprint size of 0.5 km and the large data volume for SSH fields on the oversampled 0.25 km $\times$ 0.25 km grid, the baseline plan for the SWOT Project is to make a lower-resolution and lower-noise product available on a 2 km $\times$ 2 km grid with ground-based post-processing of SWOT data to achieve a footprint size of 2 km (Rodríguez and Callahan, 2018). Since there are 16 measurements with 0.5-km footprint in each 2 km $\times$ 2 km grid cell, the variance of the uncorrelated errors with a footprint size of 2 km is a factor-of-16 smaller than (6),

$$\sigma^2_{2\mathrm{km}} = \frac{1}{16}\,\sigma^2_{0.5\mathrm{km}} = 1.875\ \mathrm{cm}^2. \tag{9}$$

The standard deviation of the uncorrelated errors with a footprint size of 2 km is therefore

$$\sigma_{2\mathrm{km}} = 1.37\ \mathrm{cm}. \tag{10}$$

The constant white noise spectrum of these uncorrelated errors for the lower Nyquist wavenumber of $l_{\mathcal{N}} = 0.25$ cpkm associated with the grid spacing of $\Delta y = 2$ km can be obtained from (9) by again inverting an expression analogous to (4) to get

$$S_{2\mathrm{km}}(l) = \frac{1}{l_{\mathcal{N}}}\,\sigma^2_{2\mathrm{km}} = 7.5\ \mathrm{cm}^2/\mathrm{cpkm}. \tag{11}$$

This spectrum is shown by the purple dashed line in Fig. 1.

Example maps of error-free SSH and noisy SSH from simulated SWOT data with uncorrelated measurement errors are shown in Fig. 4 for measurement footprint sizes of 0.5 km, 1.0 km and 2.0 km with the associated standard deviations of $\sigma_{0.5\mathrm{km}} = 5.48$ cm, $\sigma_{1\mathrm{km}} = 2.74$ cm and $\sigma_{2\mathrm{km}} = 1.37$ cm that were derived above.

It was previously noted that the complexity of validating the in-orbit performance of SWOT from the spectrum (2) of SSH after 15-km smoothing in ground-based post-processing can be avoided by instead validating the SWOT performance from the spectrum (3) of pre-processed SWOT data with 1-km footprint. Likewise, the SWOT performance can be easily validated from the spectrum (8) of pre-processed SWOT data with a footprint size of 0.5 km, or from the spectrum (11) of ground-based post-processed SWOT data with a footprint size of 2 km. As noted above, these alternative procedures for evaluating the SWOT performance assume that the spectra of the uncorrelated measurement errors are white for all wavenumbers up to the Nyquist wavenumber $l_{\mathcal{N}}$ associated with each of the three footprint sizes considered here.

The discussion of errors in this note has focused on the effects of footprint size on the standard deviation and wavenumber spectral content of the uncorrelated errors of SWOT measurements of SSH. Many SWOT users will be interested in estimates of surface velocity and relative vorticity computed geostrophically from the gridded fields of SSH. This requires spatial differentiation of SSH based on derivatives estimated by centered differences of the discrete SSH values. Each successive centered difference amplifies the errors in the SSH measurements. The standard deviations and wavenumber spectral characteristics of the resulting errors of SWOT estimates of surface geostrophic velocity and relative vorticity as functions of the smoothing applied to SWOT data in ground-based post-processing are quantified in Chelton et al. (2019; see Figs. 12–14 and the discussion in Section 4 and Appendices G and I).

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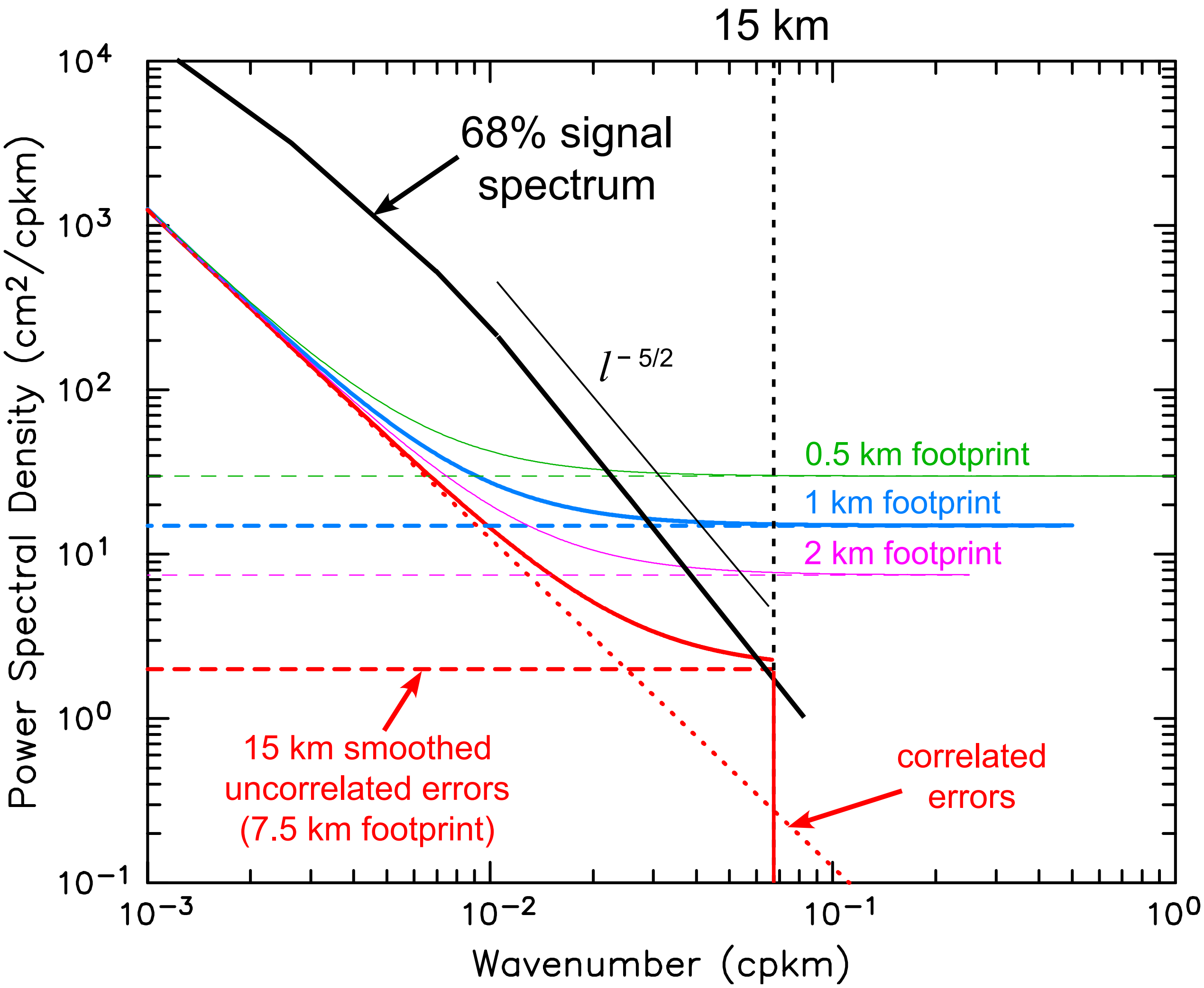

Figure 1. The 1-sided, 1-dimensional along-track wavenumber power spectral densities derived from the science requirements as discussed in the text. The dotted red line corresponds to the requirement Eq. (1) for red noise from orbit errors and long-wavelength measurement errors. The dashed red line corresponds to the requirement Eq. (2) for residual uncorrelated errors after smoothing either two-dimensionally with an ideal filter that has a magnitude of 1 for wavelengths longer than a half-power filter cutoff wavelength of $\lambda_c = 15$ km and 0 for shorter wavelengths, or smoothing only one-dimensionally in the across-track dimension with any realizable low-pass filter that has a half-power filter cutoff wavelength of λ_c. The solid red line is the sum of the power spectral densities of the red noise and the 15-km filtered uncorrelated errors. (Note that white noise smoothed isotropically with a 15-km filter cutoff is effectively equivalent to smoothing the SWOT data to a footprint size of 7.5 km.) The dashed blue line corresponds to the spectrum Eq. (3) derived here from Eq. (2) for the uncorrelated errors in pre-processed SWOT data for a footprint size of 1 km without the 15-km smoothing. The solid blue line is the spectrum of the total errors, i.e., the sum of the red noise and the white noise with 1-km footprint size. The green and purple lines are the analogous spectra of uncorrelated errors and total errors for the footprint sizes of 0.5 km (green) and 2 km (purple) that are discussed at the end of this document, shown for wavenumbers up to the associated Nyquist wavenumbers of 1 cpkm and 0.25 cpkm for, respectively, the sample grids of 0.5 km and 2 km on which the white noise is uncorrelated for these two footprint sizes. The thick black line is the 68$^{\text{th}}$-percentile SSH signal power spectral density from the SWOT Science Requirements Document (Rodriguez and Callahan, 2016), which is based on analysis of Jason altimeter data down to a wavelength of 70 km with extrapolation to shorter wavelengths. For reference, a power-law rolloff of $l^{-5/2}$ for alongshore wavenumber l is shown as the thin black line. (Figure adapted from Fig. F.2 of Chelton et al., 2019.)

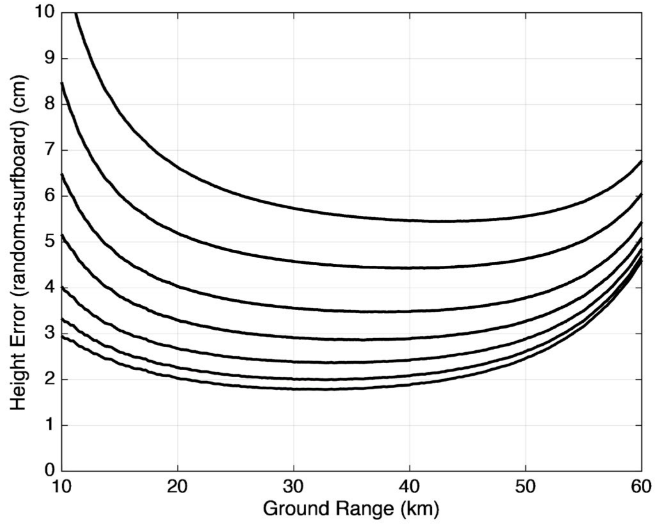

Figure 2. The current projected estimates of the dependencies of the standard deviation of uncorrelated errors of SWOT measurements of SSH on significant wave height (SWH) and swath location relative to the satellite ground track for a footprint size of 1 km. The seven solid lines correspond to SWH values increasing from 2 m to 8 m at increments of 1 m (bottom to top). The errors for 0-m SWH are essentially the same as those shown for 2-m SWH. In accord with Eqs. (5), (7) and (10), the standard deviations of the uncorrelated errors are a factor-of-2 larger for a footprint size of 0.5 km and a factor-of-2 smaller for a footprint size of 2 km. (Figure adapted from Fig. 2.5 of Gaultier et al., 2017.)

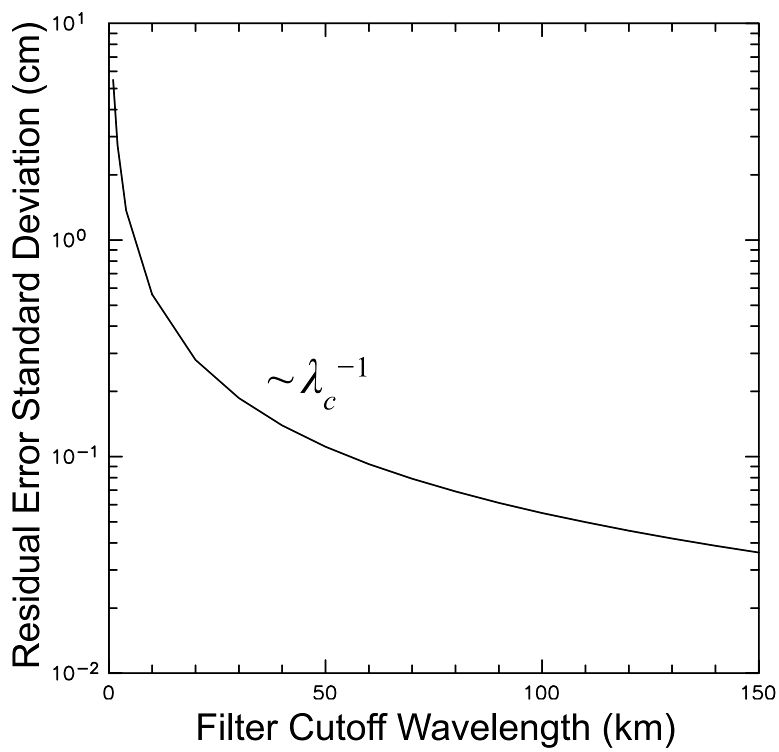

Figure 3. The standard deviations of residual uncorrelated SSH measurement errors as a function of half-power filter cutoff wavelength λ_c with isotropic 2-dimensional smoothing of SWOT data. The effective footprint diameter is half of the half-power filter cutoff wavelength (see Appendix B of Chelton et al., 2019). The first three points correspond to the error standard deviations of 5.48 cm, 2.74 cm and 1.37 cm given by Eqs. (7), (5) and (10) for footprint sizes of 0.5 km, 1 km and 2 km, respectively. The power-law dependence on λ_c that is labeled on the curve was determined from a log-log version of the figure (not shown here). (Figure adapted from Fig. 12 of Chelton et al., 2019.)

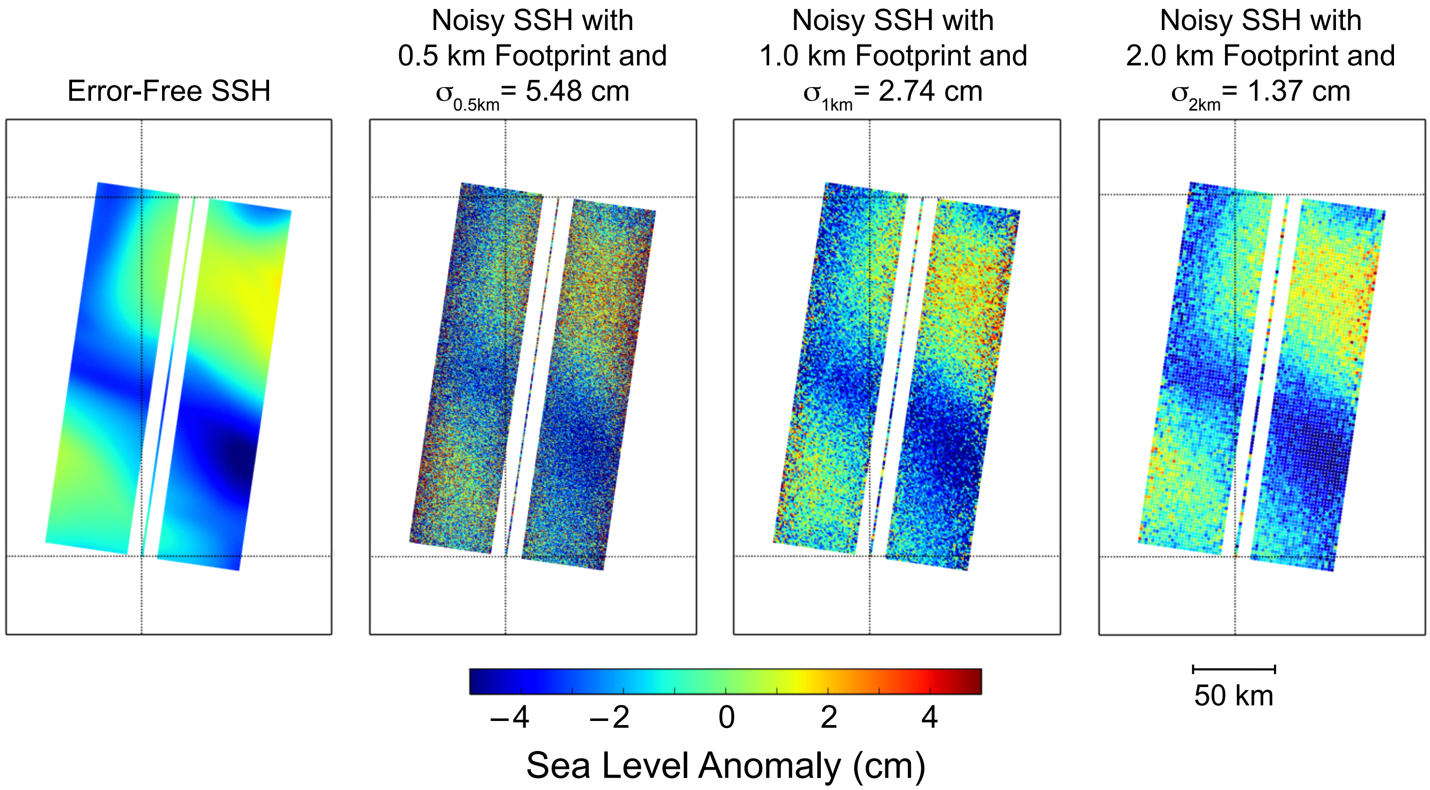

Figure 4. Example maps of error-free SSH and noisy SSH from simulated SWOT data with uncorrelated measurement errors for footprint sizes of 0.5 km, 1.0 km and 2.0 km with the associated standard deviations of $\sigma_{0.5\text{km}} = 5.48$ cm, $\sigma_{1\text{km}} = 2.74$ cm and $\sigma_{2\text{km}} = 1.37$ cm derived in the text. Actual SWOT data will also be contaminated by long-wavelength measurement errors. The scale for a distance of 50 km is labeled. (Figure courtesy of Nathalie Steunou.)